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Kelly, Roger W.; Nelson, Frank C.

Monterey, California: U.S. Naval Postgraduate School

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A STATISTICAL EXAMINATION OF THE 500-MB 48-HR PROGNOSES PREPARED BY FLEET NUMERICAL WEATHER FACILITY, MONTEREY, CALIFORNIA.

ROGER W. KELLY AND
FRANK C. NELSON, JR.

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A STATISTICAL EXAMINATION OF THE 500-MB 48-HR PROGNOSES PREPARED BY FLEET NUMERICAL WEATHER FACILITY, MONTEREY, CALIFORNIA

* * * * *

Roger W. Kelly

and

Frank C. Nelson, Jr.

A STATISTICAL EXAMINATION OF THE 500-MB 48-HR PROGNOSES PREPARED BY FLEET NUMERICAL WEATHER FACILITY, MONTEREY, CALIFORNIA

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ABSTRACT

The Fleet Numerical Facility (FNWF) 500-mb long-wave prognoses at latitudes 20N through 70N were analyzed statistically for possible error-bias. FNWF issued a correction field (verified minus prognostic) for each latitude circle and day under study as well as the initial height field 48 hours earlier.

Fourier analyses of both the correction fields and the initial height fields were made. Spectral analyses of the correction fields indicated that more than 60% of the error lay in correction-waves 1, 2, and 3 at all latitudes and that this figure increased northward.

Persistency correlations were made between the initial height fields and the correction fields. The results indicated a carry-over of persistency into the final 48-hr forecast, especially in the cases of heights which were initially considerably above or below the normal heights.

The final phase of the study dealt with the question of stabilization of the ultra-long waves (1, 2, and 3), using the present operational barotropic model. The model seems to retard waves 1 and 3 at the lower and middle latitudes, and to move them too far at high latitudes. The results for wave 2 are somewhat less conclusive, but are generally in agreement with those for waves 1 and 3.

The authors wish to express their sincere appreciation to Professor Frank L. Martin, Department of Meteorology and Oceanography, U. S. Naval Postgraduate School, for his suggestion of the topic and assistance in the preparation of this study.

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LIST OF SYMBOLS

Symbol	Definition
s	wave number (1 through 12)
r	data-point number (0 through 35)
n	number of data points per latitude (36)
α_s	coefficient of cosine term of Fourier series (sth wave)
β_s	coefficient of sine term of Fourier series (sth wave)
C_s or A_s	amplitude of correction field (sth wave)
$C_s^2/2$	explained variance of correction field by sth wave
Ψ_s	phase angle of sth wave
$\Phi_s = \Psi/s$	"phase shift" of sth wave
Φ_c	phase shift of correction field (verification day)
$\Phi_{z,-2}$	phase shift of analyzed height field (forecast day; two days prior to verification)
$\Delta\Phi = \Phi_c - \Phi_{z,-2}$	relative phase shift
$\lvert \Delta\Phi \rvert$	magnitude of relative phase shift
$Y(r)$	value of a correction field or a height field by latitude as a function of meridian r

1. Introduction.

Fleet Numerical Weather Facility (henceforth referred to as FNWF) issues a 48-hr "long wave" prognosis for 500 mb. This forecast is issued twice daily (00Z and 12Z) and is based upon a smoothed version of their conventional 48-hr map. The smoothing is performed by means of the operator

$$\overline{Z}_o = \left[1/8 \left(Z_1 + Z_2 + Z_3 + Z_4 \right) + 1/2 \; Z_o \right]^4$$

which reduces the amplitudes of the short waves while leaving that of the long waves relatively unchanged.

FNWF provided the authors with 36 daily values of the forecast and verifying heights at 10^o longitude intervals along each of the six latitude circles 20 through 70N. In addition, they also provided for each case similar data from the analyzed chart which served as the initial-data chart 48 hours earlier. The authors then examined, by statistical means, the hypothesis that the correction field (i.e., the set of correction values, as a function of latitude L and longitude r, necessary to convert the forecast long-wave chart to the verifying chart) was at least partially determined or biased by the original analysis used in starting the 500-mb prognosis.

2. Data Sources.

As mentioned in the previous section, the primary data-charts used in this study were the analyzed 500-mb long-wave charts at 0000 GCT together with the "correction" long-wave charts timed 48 hours later. The data were restricted to the month of December 1962, so that it was possible to make a total of 29 comparisons between initial and correction charts.

3. Statistical Techniques.

(a) Spectral Analysis of the Correction Field

Before any relationships between the correction and initial fields were sought, it was decided to investigate the spectral characteristics of the correction field. This was done by expressing the "correction" long-wave field, denoted $Y(r)$, as the sum of twelve harmonic waves by Fourier analysis:

$$Y(r) = 1/2\,\alpha_0 + \sum_{s=1}^{12}\left(\alpha_s \cos\frac{2\pi rs}{n} + \beta_s \sin\frac{2\pi rs}{n}\right) \qquad (1)$$

where s = wave number (1 through 12).

n = 36 is the number of data points on each latitude circle

r = 0, 1, 2, ..., n -1 is a distance scale numbered in equally-spaced data points measured westward from Greenwich.

Then the spectral contribution of the sth harmonic in $Y(r)$ is given by

$$1/2\left(\alpha_s^2 + \beta_s^2\right) \text{ or } 1/2\ C_s^2 \qquad (2)$$

The last statement implies that (2) has been expressed in the equivalent form

$$Y(r) = 1/2\,\alpha_0 + C_s \cos\left(\frac{2\pi rs}{n} - \Psi_s\right) \qquad (3)$$

where Ψ_s is the phase angle. It has been found more convenient in this study to use phase "shift" Φ_s rather than phase angle. This is done by expressing $Y(r)$ as

$$Y(r) = 1/2\,\alpha_0 + C_s \cos\left[s\left(\frac{2\pi r}{n} - \Phi_s\right)\right] \qquad (4)$$

The advantage of using (4) over (3) is that it enables one to keep track of the same identifiable feature (for example, the ridge

closest to Greenwich) for each wave number .

Using the CDC-1604 digital computer, the Fourier series was derived for each latitude and day. Use was made of the FORTRAN program "Periodogram Analysis" written by R. R. Hilleary [3] of the U. S. Naval Postgraduate School Computer Center staff. The program-output gave for each wave number S, each latitude and day under consideration, the following:

(1) Amplitude C_s and spectral value $C_s^2/2$

(2) Phase angle Ψ_s and phase shift $\Phi_s = \Psi/s$

(3) Mean value of Y and the total variance of $Y(r)$

(4) Percent contribution of the sth wave to total variance of $Y(r)$

In this subsection, attention was focused upon item 4, that is, relative spectral contributions of the various correction-field waves. In order to obtain representative values, the 29 daily percent contributions by waves were averaged to give the December 1962 mean relative-percentage contributions to variance of the correction field. The resulting percentages are displayed in table 1, both as a function of latitude and wave number.

In addition, any wave listed in table 1 which contributed, in the mean, more than 10% of the variance at its indicated latitude has been subjected to frequency analysis using histograms. The waves so examined are indicated in table 1 by an asterisk following the appropriate percentage contribution. Waves 1 to 4 fall into this category at latitudes 20 through 50, while waves 1 to 3 contribute more than 10% at all latitudes 20 through 70. Wave 6 contributes more than 10% at latitude 60, but all other values of all waves fall below 10%.

Table 1. Mean explained percentage contribution to the total variance of the correction field by wave number and latitude.

	20N	30N	40N	50N	60N	70N
Wave No. 1	14.5*	21.2*	20.9*	16.7*	30.3*	35.2*
Wave No. 2	17.7*	16.9*	19.9*	25.3*	26.6*	32.2*
Wave No. 3	28.9*	22.1	18.9*	14.5*	15.9*	16.2*
Wave No. 4	16.6*	12.8*	12.1*	10.9*	8.1	9.5
Wave No. 5	6.5	6.2	5.9	6.8	8.4	3.3
Wave No. 6	3.9	4.7	7.2	10.1*	4.7	1.3
Wave No. 7	3.3	4.5	4.9	7.3	2.8	1.3
Wave No. 8	2.0	2.9	3.0	2.3	1.3	0.4
Wave No. 9	1.8	3.2	2.6	1.8	0.6	0.3
Wave No. 10	1.5	2.1	1.4	0.9	0.4	0.1
Wave No. 11	0.7	1.4	1.2	0.9	0.3	0.1
Wave No. 12	0.9	0.7	0.7	0.7	0.1	0.0
Total Explained Variance	98.3	98.7	98.7	98.2	99.5	99.9

* Denotes wave contributing more than 10% to the correction field at the indicated latitude.

The histograms drawn are shown in the Appendix as figures 1-6 (in order of increasing latitude) for wave 1, figures 7-12 for wave 2, figures 13-18 for wave 3, figures 19-22 for wave 4, and figure 23 for wave 6. Note that all these diagrams present in the upper half a frequency distribution with regard to wave amplitude, and in the lower half a frequency distribution of the phase shift giving the correction-wave ridge closest to Greenwich.

As an example of the type of conclusion which may be drawn from these histograms, one may compare figure 1 with figure 6, both dealing with wave 1. Figure 1 indicates that wave 1 has a small mean amplitude (66 feet) and a mean ridge position at -15W longitude*. By contrast, figure 6 has a mean amplitude of 155 feet and a mean correction phase shift of -7W longitude*. However, note that frequency distributions of the phase shift appear to be random in both cases, though not a Gaussian distribution in either case.

The same general latitudinal variation occurs with wave 2 in passing from latitudes 20N to 70N. At 20N there is only a small mean amplitude (about 60 feet) and a random distribution of phase shifts centered around a mean of 50W, whereas at latitude 70N the wave 2 correction field had an amplitude of 150 feet and a phase shift of 15W. This means that the trough in the wave 2 correction field at 70N would be located at 105W in December.

The mean values of amplitudes and phase shifts in figures 1-23 have been computed and are displayed by each histogram. It is not known whether such means have any real significance. Of course it is possible that the correction field histograms depicted show real prognostic anomalies which the barotropic forecast model cannot resolve, and are applicable to December in general. One approach in determining whether these correction-waves have real statistical significance is to test the cross-correlation between correction and harmonic waves by latitude and wave number

$$r\left(c_s\ ,\ z_{s,-2}\right) = \frac{c_s\ c_{z,-2}\ \cos\left(\Phi_c - \Phi_{z,-2}\right)}{\sigma_{c_s}\ \sigma_{z_s}} \qquad (5)$$

* Note that all FNWF long-wave information was printed in terms of degrees west longitude so that the two phases mentioned above are 15° and 7° east longitude.

for significance. The expression on the right side of (5) is given by Kahn [4]. Actually this test was not performed, mainly because a simpler approach, described below, was applied.

(b) Correlation Investigation of Anomalies

A FORTRAN data-sorting program was written for each latitude and all of the 29 days. This program sorted out abnormally high and low contour values in the 29-day sample, and a correlation subroutine computed the correlation between the initial contour height at the latitude-longitude intersection (L, r) and the forecast-correction value verifying two days later. The sorting program was designed to divide the 29 x 36 value sample at each latitude into classes as follows:

(I) Those heights more than one standard deviation above the sample mean height (approximately 16% if the heights are normally distributed).

(II) Those heights smaller than the mean by at least one standard deviation.

(III) The remaining 68% of the heights between extremes (I) and (II).

The linear correlations obtained for each latitude and class are listed in table 2 below.

Table 2. Linear correlations between initial height fields and 48-hr verification-correction values.

	20N	30N	40N	50N	60N	70N
Class I	0.08	-0.01	0.02	0.16	0 06	-0.41
Class II	-0.29	-0.17	-0.13	0.09	0.15	-0.22
Class III	-0.03	-0.13	0.07	-0.12	-0.03	-0.13

If we make the assumption that half of the data-sample pairs are uncorrelated, significance tests for each class will be based upon the

reduced sample sizes N = 84, 84, and 354, respectively, for classes I, II and III. For these sample sizes, the following critical values r_c of the linear correlation coefficient are applicable (see Dixon and Massey [1957, p. 468] at the 99% confidence level):

(I) N = 84 $r_c = 0.27$

(II) N = 84 $r_c = 0.27$

(III) N = 354 $r_c = 0.13$

A significant negative correlation between correction values and the 500-mb heights two days earlier indicates that initially above-average heights tend to perpetuate themselves unduly in the prognoses two days later. This effect of persistence is most marked with classes I and II, which are already considerably above or below normal. The most significant correlations in these classes correspond to (I, 70N) and (II, 20N) with $r = -0.41$ and -0.29, respectively. Generally, class II shows a preponderance of near-significant negative values (see latitudes 30, 40, and 70N in addition to latitude 20N), which tend to bear out the comments made previously with respect to the effects of persistence.

Conversely, class III, even with the large number of "independent" pairs, has no definitely significant correlation coefficients. These results suggest that the FNWF barotropic model appears to handle weak ridges and troughs quite well.

(c) Persistence of the Ultra-Long Waves

As mentioned in section 3(a), the frequency distributions of correction-wave phase shifts Φ_c shown in figures 1-23 cannot be expected to apply for another December forecast series. Moreover, while the frequency distributions do not appear to be normally distributed,

there is an element of randomness about them in terms of values of Φ_c relative to Greenwich. Hence, a relative test concerning the efficacy of the correction-wave phase shift Φ_c has been devised. In this test, attention is focused only on the harmonic waves $S = 1, 2, 3$, called "ultra-long" waves.

The barotropic model employed by FNWF makes use of a long-wave "stabilization" term called the Helmholtz correction term. This term is due to the initial work of Wolff [6] and Cressman [7], and was devised specifically to keep waves $S = 1, 2, 3$ stationary, whereas the previous version of the barotropic model permitted spurious movement of these waves. For the purpose of testing whether these waves were stabilized, the variable

$$\Delta\Phi = \Phi_c - \Phi_{z_j-2} \qquad (6)$$

which may be termed "relative phase shift," was computed at each latitude for each day and wave numbers $S = 1, 2, 3$. In (6) the symbol Φ_{z_j-2} represents the phase of the Sth Fourier wave derived from the analysis two days previous to the long-wave prognosis. In (6) both Φ_c and Φ_{z_j-2} denote the meridians of the forecast-correction and initial waves closest to the Greenwich meridian. It was considered, in this section of the work, that if $\Delta\Phi$ lay in the range -5 to +5 for the 48-hr forecast period, the wave was actually stabilized and such values of $\Delta\Phi$ were termed "zero" for the purposes of this investigation.

Values of $|\Delta\Phi|$ larger than 5° longitude were considered to be exact, and the number of positive, negative and "zero" values occurring in each sample of 29 cases are listed in table 3 for each latitude. In addition, mean values of $\Delta\Phi$ and mean magnitudes $|\Delta\Phi|$ have been listed.

Table 3. Some statistics on the behavior of relative phase shift.

	Latitude	Mean $\Delta\Phi$	Mean $\lvert\Delta\Phi\rvert$	Signs of $\Delta\Phi$ Positive	Negative	Zero	Significance Level
Wave No. 1	20N	-12.9°	120.4°	17	12	0	*
	30N	-71.3°	104.7°	6	22	1	0.2%
	40N	-64.1°	76.4°	2	25	1	0.2%
	50N	-57.6°	77.4°	6	23	0	0.2%
	60N	+ 6.5°	116.3°	13	15	1	N
	70N	+19.0°	104.7°	14	15	0	N
Wave No. 2	20N	+18.7°	32.3°	22	5	2	*
	30N	- 7.5°	44.5°	11	14	4	N
	40N	0.0°	34.8°	11	15	3	N
	50N	- 6.9°	32.7°	12	16	1	N
	60N	-23.4°	50.1°	8	20	1	1.2%
	70N	- 3.2°	47.5°	13	14	2	N
Wave No. 3	20N	+13.6°	49.2°	17	10	2	*
	30N	-13.6°	37.6°	8	19	2	2.6%
	40N	-15.6°	28.8°	6	22	1	0.2%
	50N	+ 1.5°	25.8°	13	14	2	N
	60N	+ 5.8°	29.9°	12	13	4	N
	70N	+ 5.9°	29.9°	17	10	2	12.4%

* Latitude 20N not included in test.

N Indicates the distribution of signs is not significantly different from a "chance" distribution.

The first of these gives a ready answer to the question of the 48-hr displacement of the ultra-long correction-waves in degrees west longitude. From table 3a, it appears (see column 1, excluding latitude 20N) that:

Wave 1 correction field is over-displaced to the east in latitudes 30-50N, but is displaced westward at 60 and 70N.

Wave 2 is slightly over-displaced at all latitudes.

Wave 3 is over-displaced at low latitudes (30, 40N), but negatively displaced at high latitudes.

Significance levels have not been ascribed to mean $\Delta\Phi$ values in table 3, so that the conclusions on displacement proposed just above are only tentative and subject to further testing, for example using the t-test.



A test has also been applied to the signs contained in the sign column of table 3. The significance levels have been determined (using table 10a, p 418 of Dixon and Massey, 1957) and the implication is that, at these levels, the combination of positive and negative signs could only occur with a probability equal to the quoted level in table 3, assuming either type of sign were equally probable.

The conclusions to be drawn from the sign-significance tests as applied to table 3 are as follows:

(a) For wave 1 there is strong evidence that the ultra-long waves require displacement to the east relative to the prognostic location at latitudes 30, 40, and 50N in a majority of the cases. This result is in agreement with that noted earlier in connection with the value of the mean $\Delta \Phi$.

(b) For wave 2 at higher latitudes (60N) the barotropic model gives under-displacement compared to observation.

(c) For wave 3 there is a curious but consistent pattern at latitudes 30 and 40N. The barotropic model gives consistent under-displacement as compared to observation. The opposite conclusion at latitude 70N appears to have moderate significance, that is, a 12.4% "chance-probability." At latitudes 50 and 60N, the distribution of signs appears to be completely random.

4. Conclusions.

These statistical studies of the FNWF 500-mb prognoses have shown a number of significant results. Some of these results are given below.

(a) More than 60% of the variability of the 48-hr correction field is associated with correction waves 1, 2, 3 with the former taking on a consistently greater proportion with increasing latitude.

(b) There is a tendency for above and below normal heights to be perpetuated in the subsequent 48-hr forecast.

(c) Generally in the low latitudes (30 through 50N) the ultra-long waves must be displaced to the east of their prognostic location, while the reverse seems to be true at latitudes 60 and 70N.

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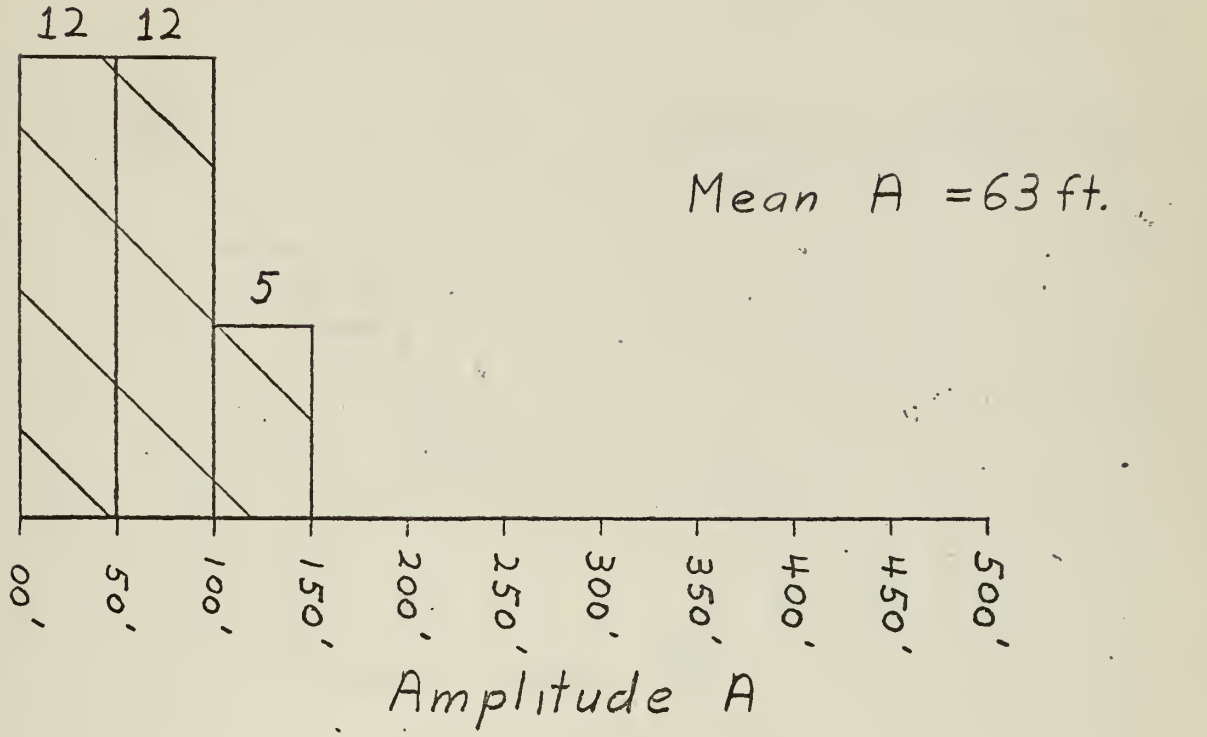
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APPENDIX

Histograms of amplitude and phase shift for correction waves accounting for more than 10% of the latitudinal variance of the wave (figures 1 through 23). For descriptions of these figures and appropriate page numbers, refer to the list of figures on page iv.

Figure 1

Wave No. 1 , Latitude 20° North



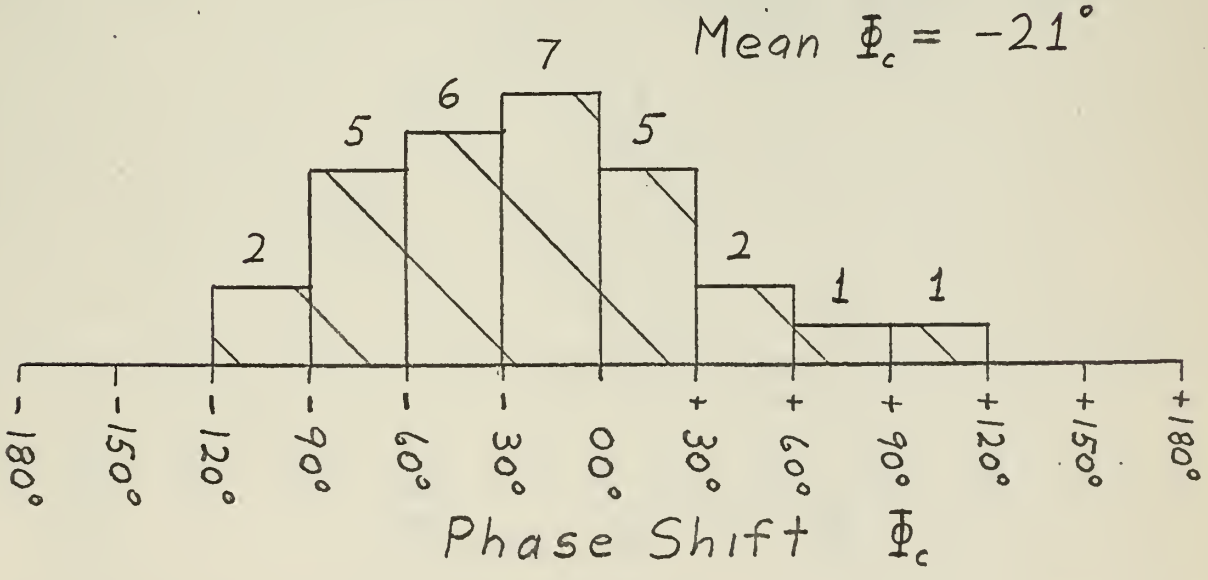
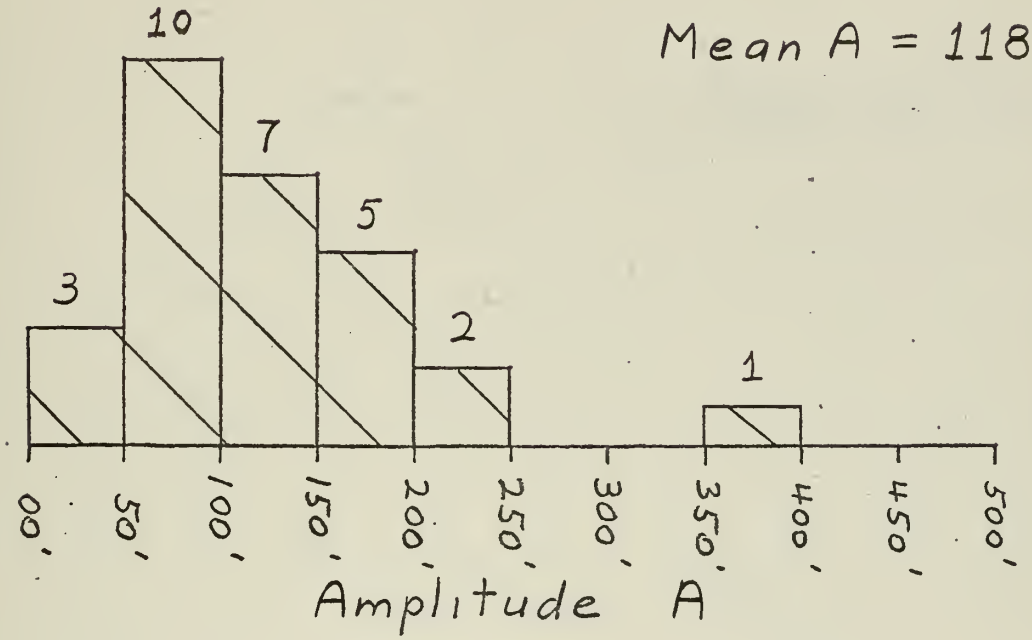




Figure 2

Wave No. 1, Latitude 30° North



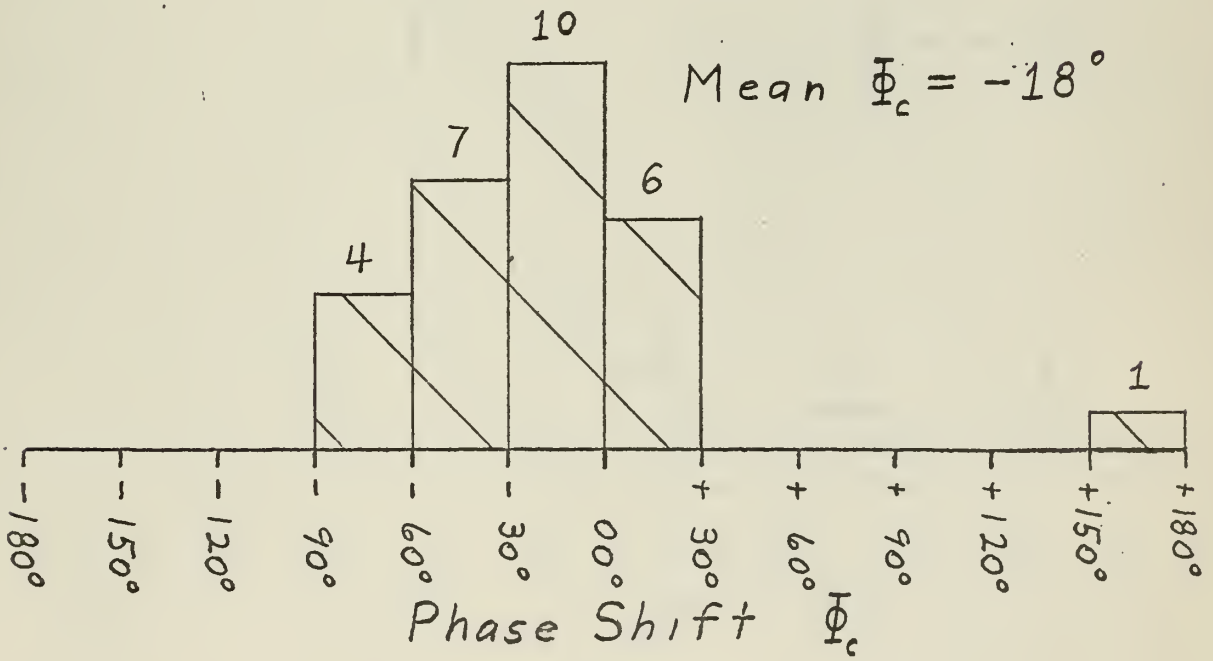




Figure 3

Wave No. 1, Latitude 40° North

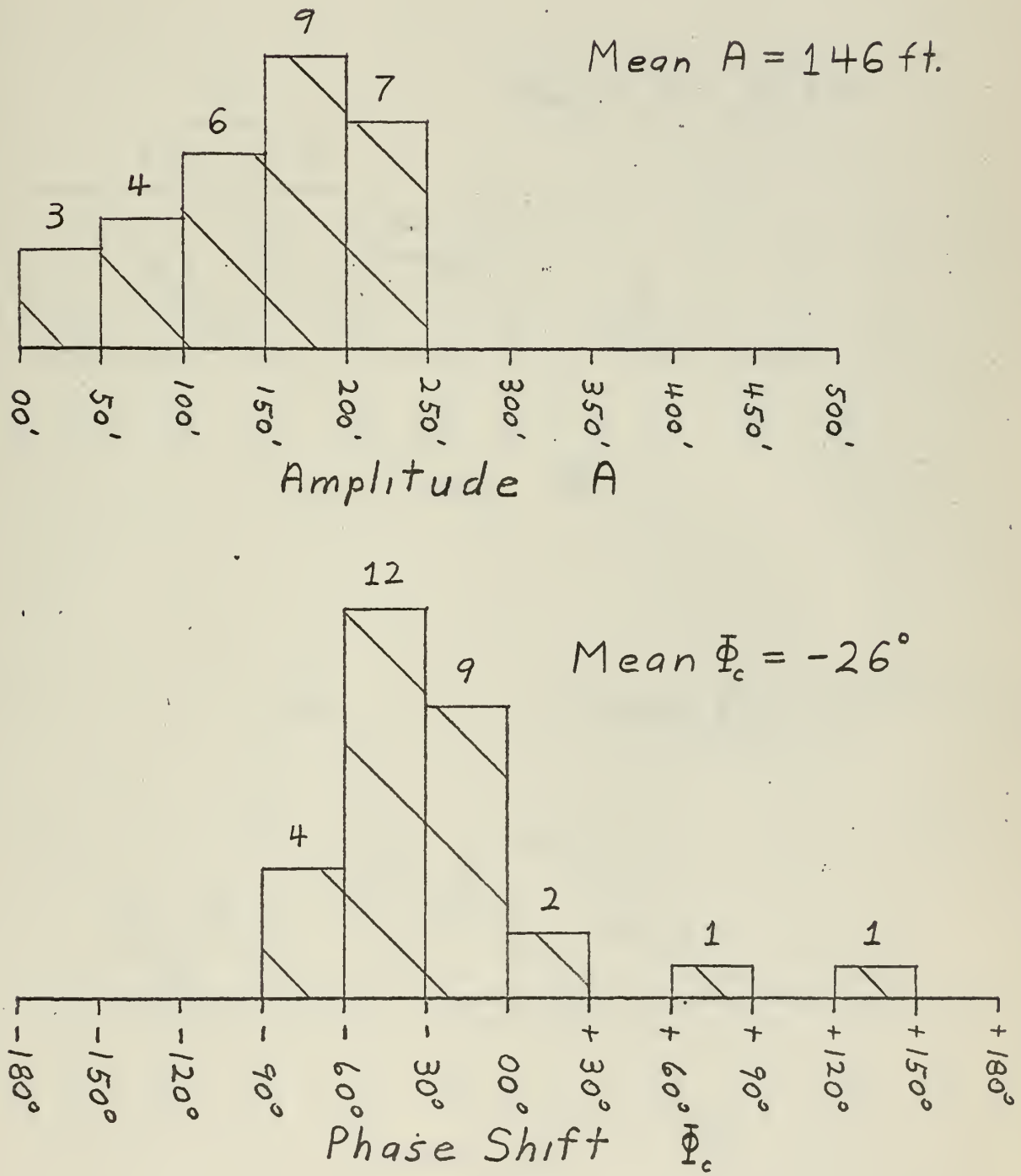
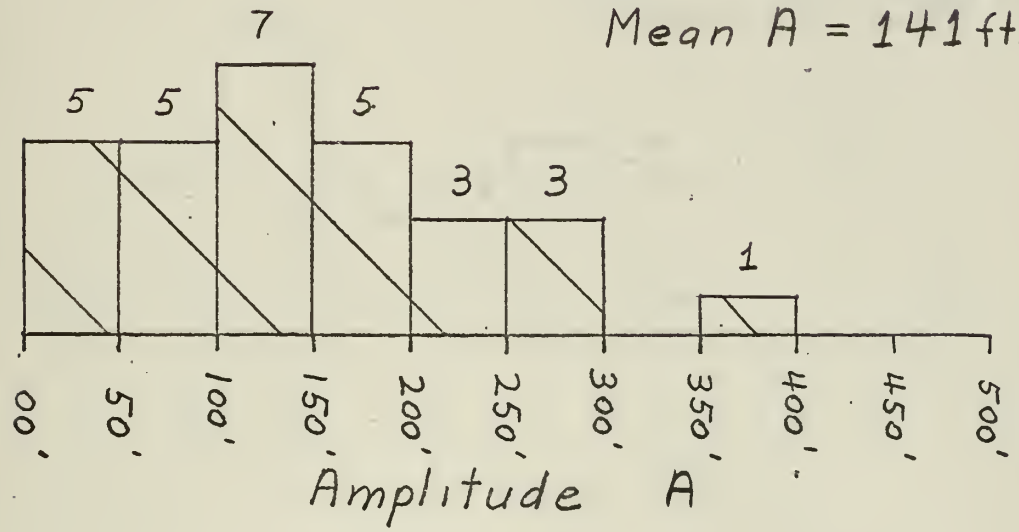




Figure 4

Wave No. 1 ; Latitude 50° North



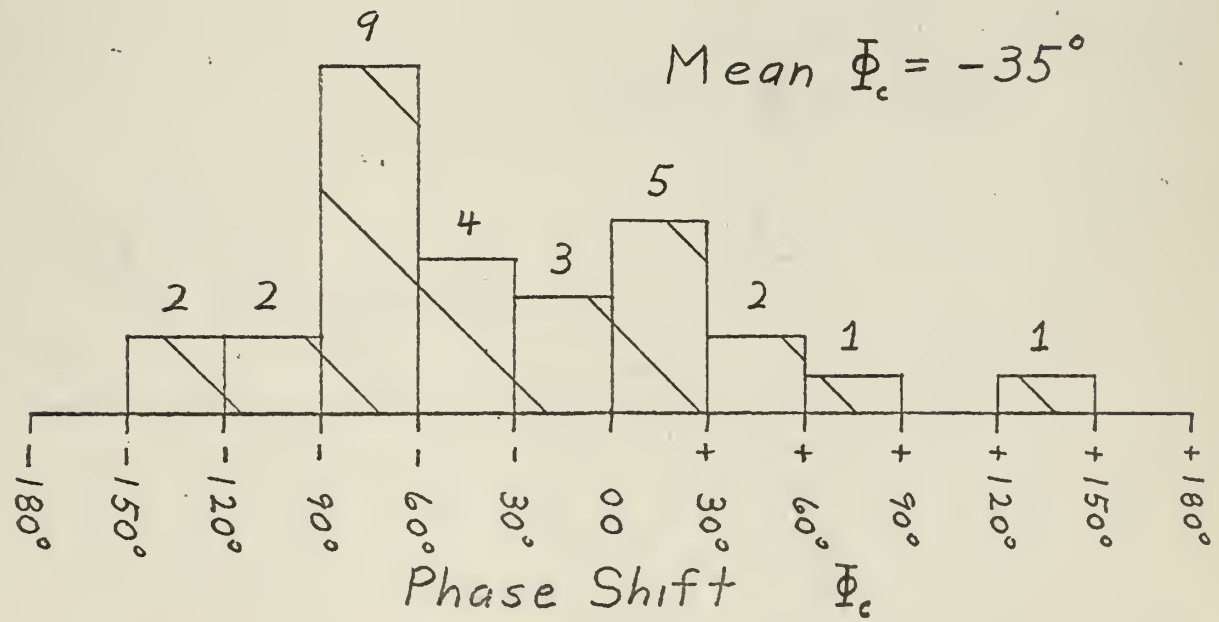
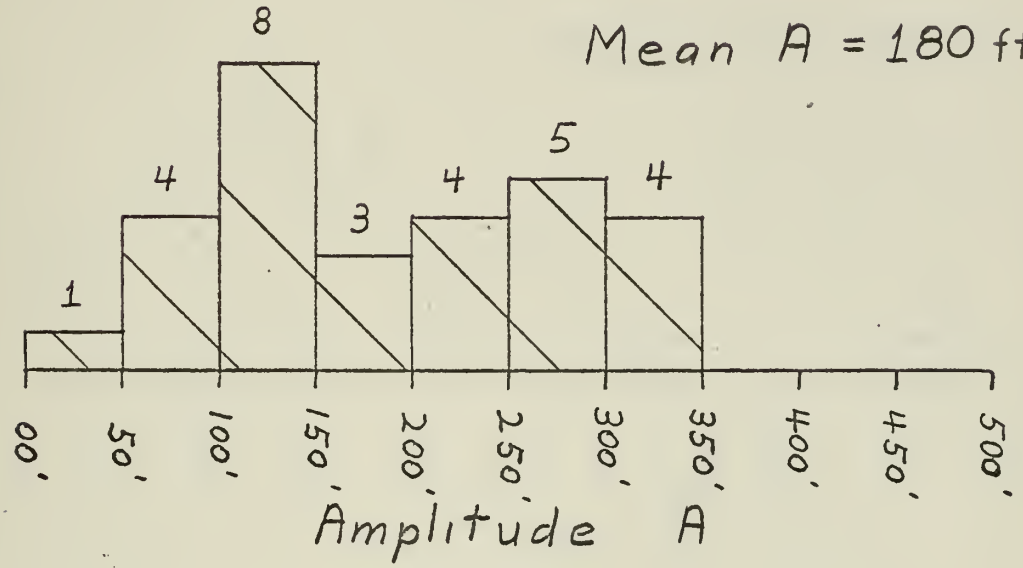
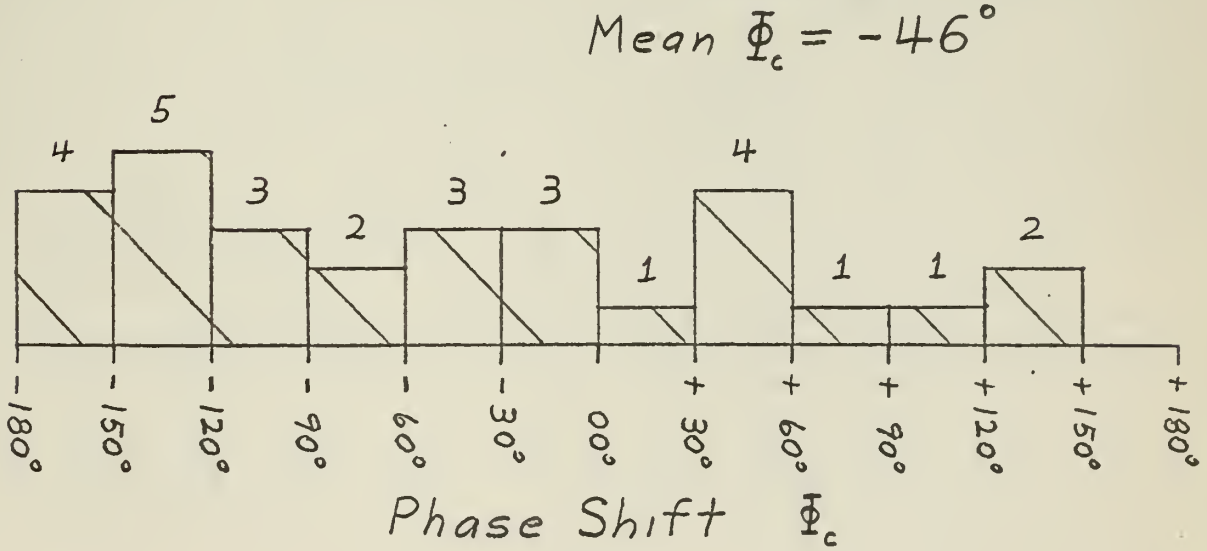




Figure 5

Wave No. 1, Latitude 60° North





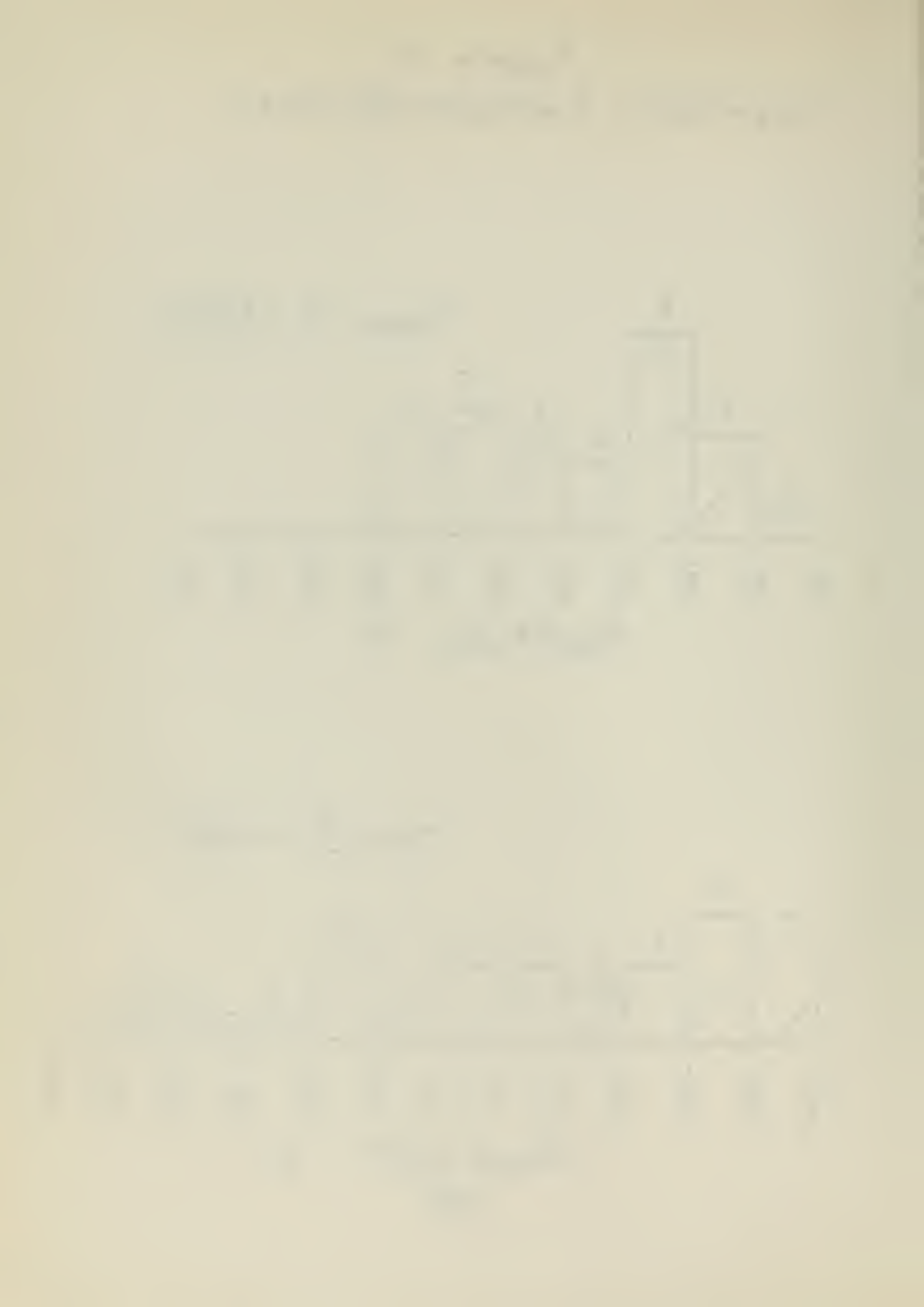
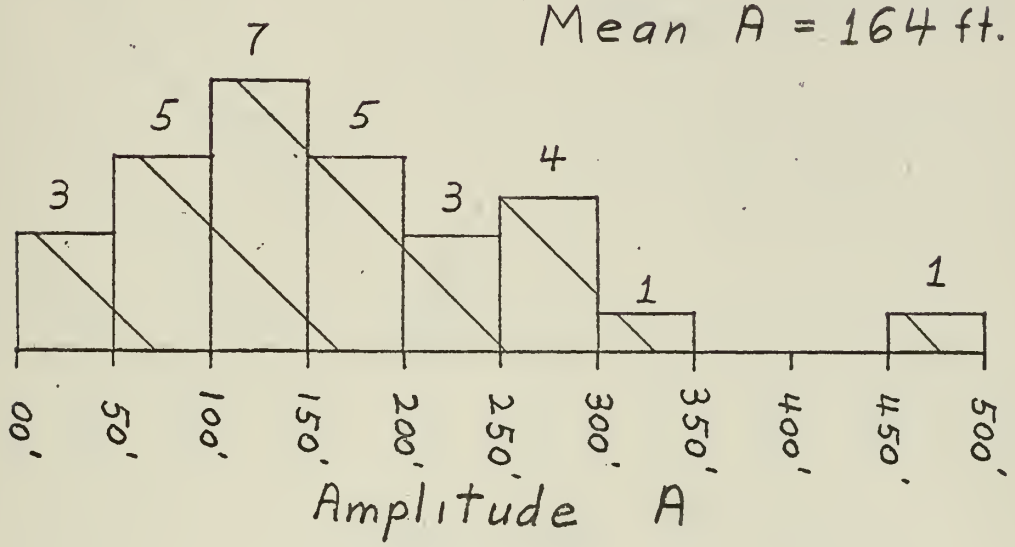
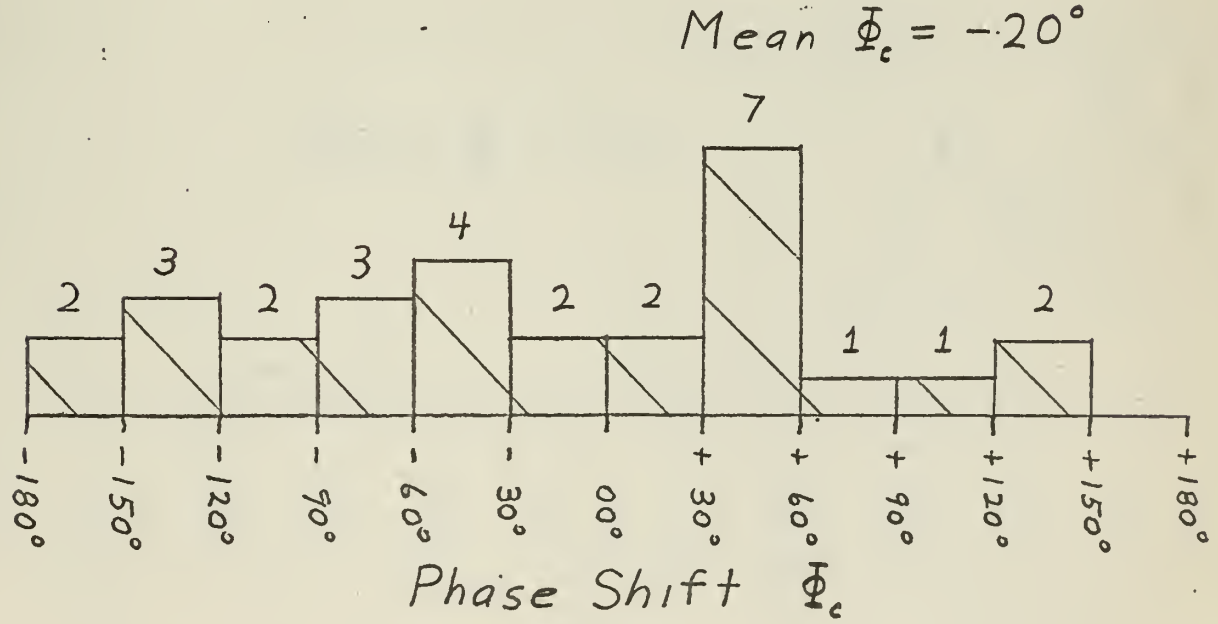


Figure 6

Wave No. 1, Latitude 70° North





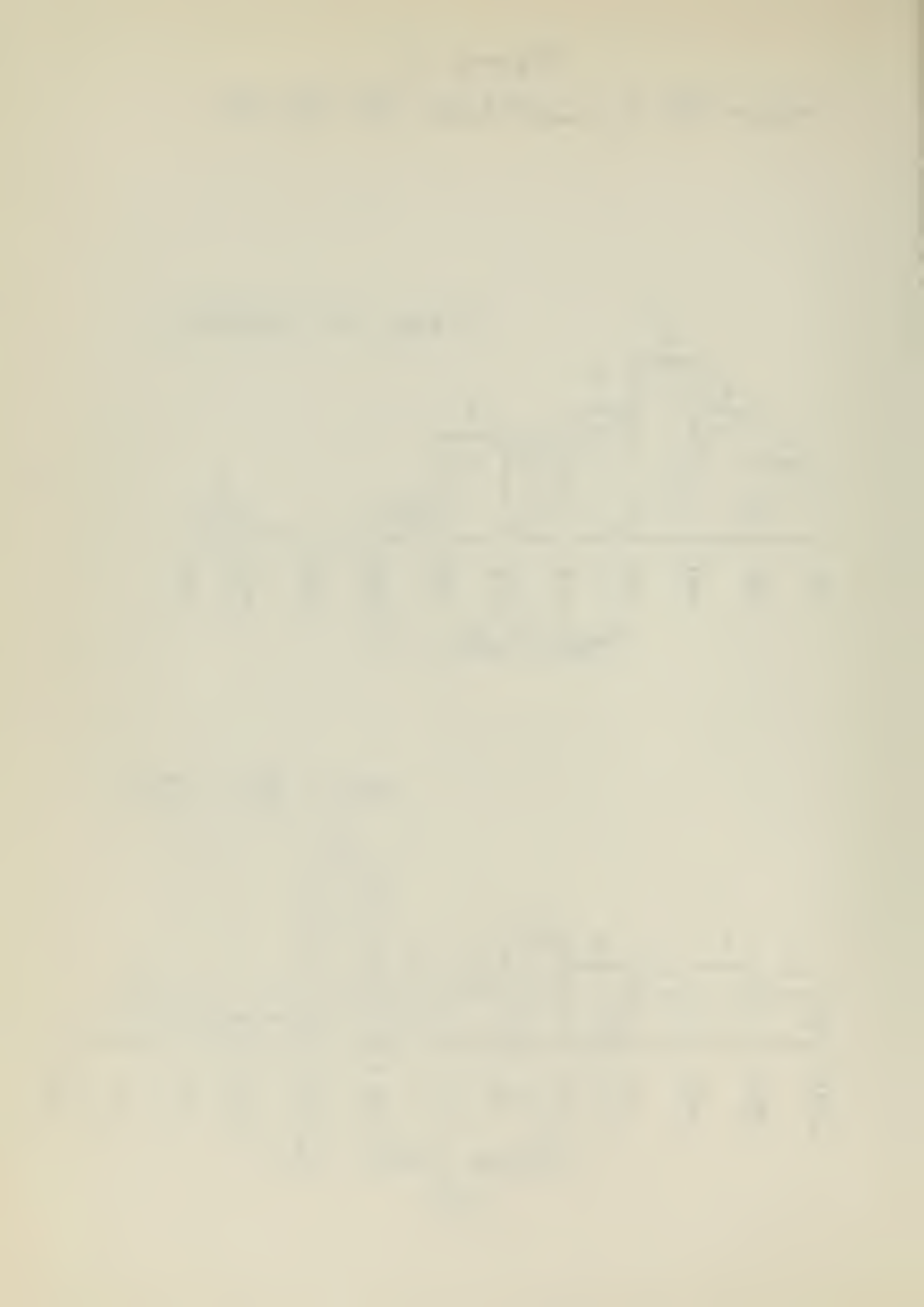
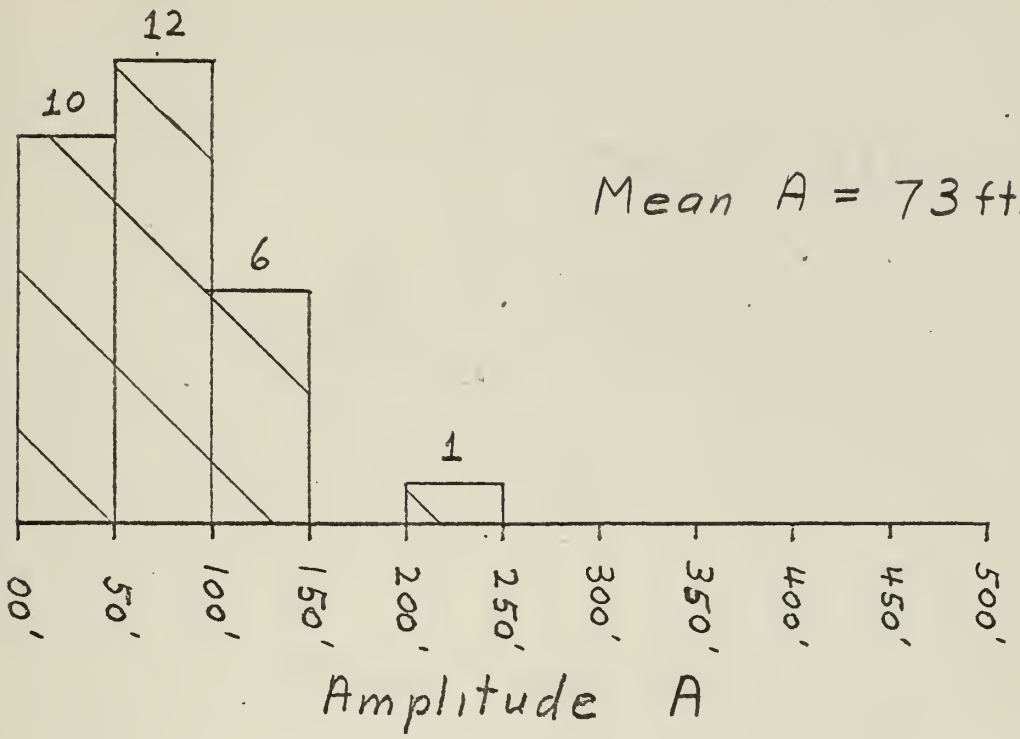


Figure 7

Wave No. 2, Latitude 20° North



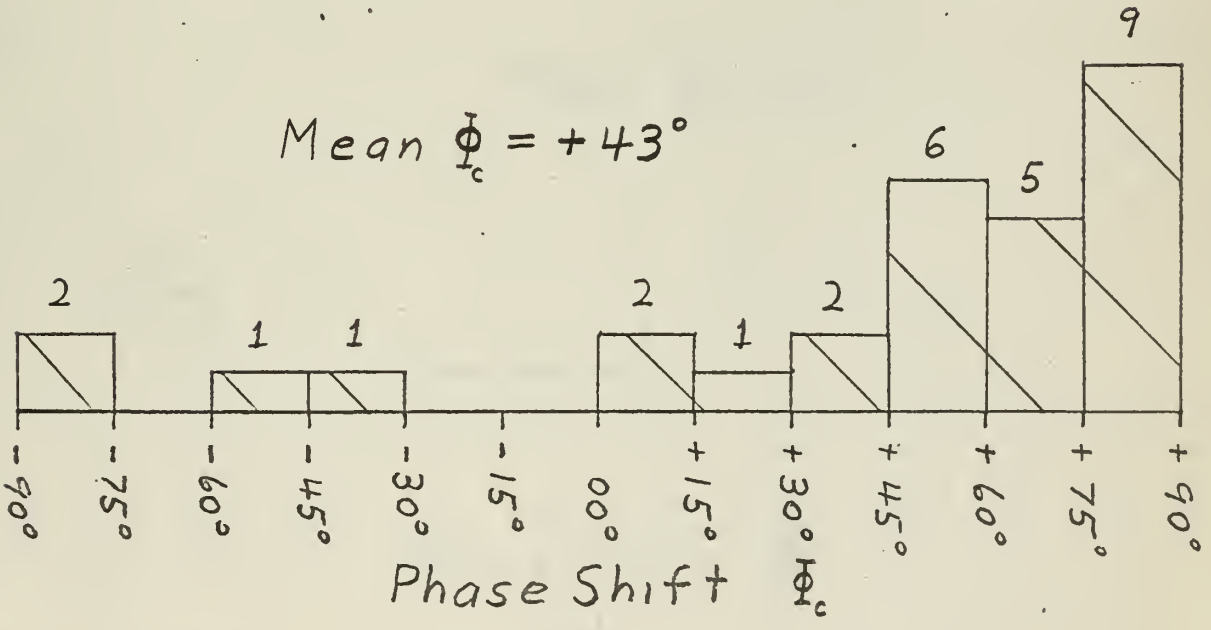
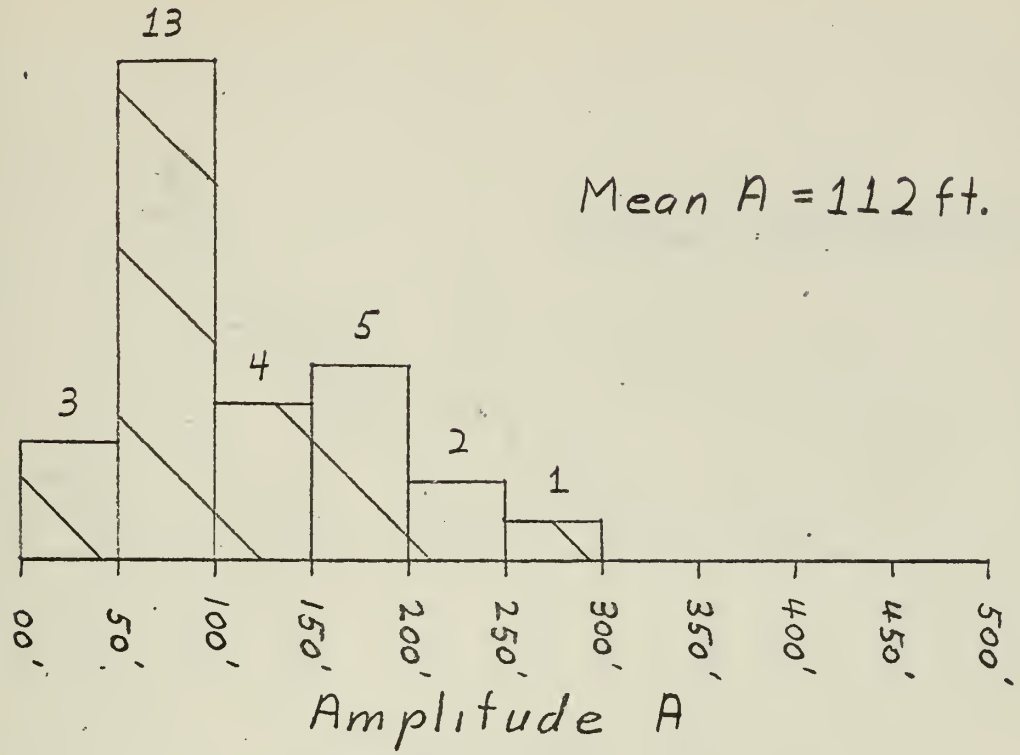




Figure 8

Wave No. 2, Latitude 30° North



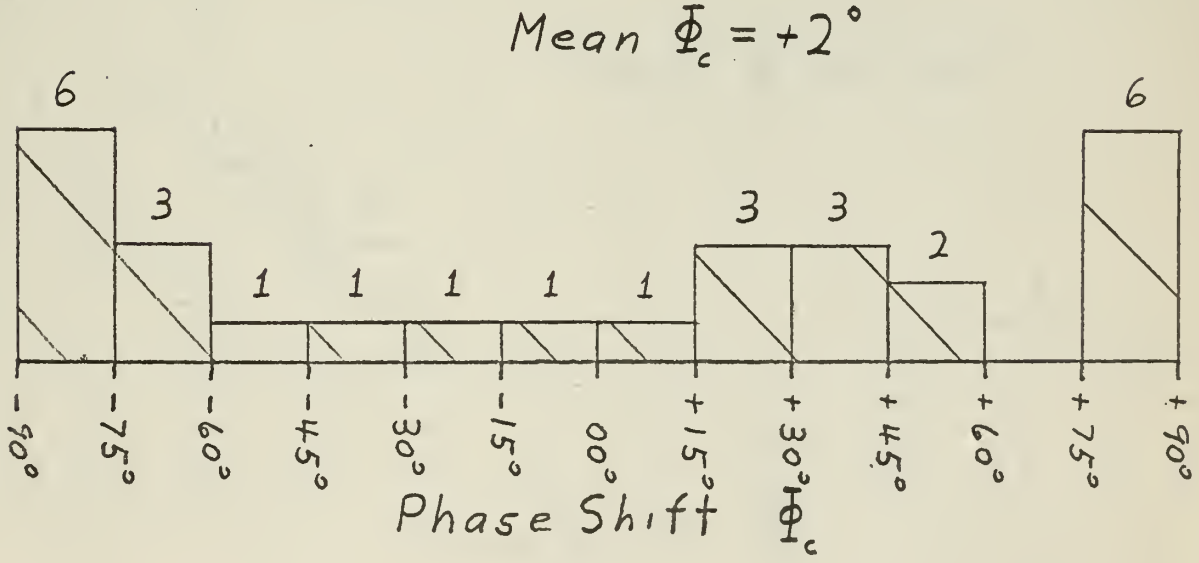
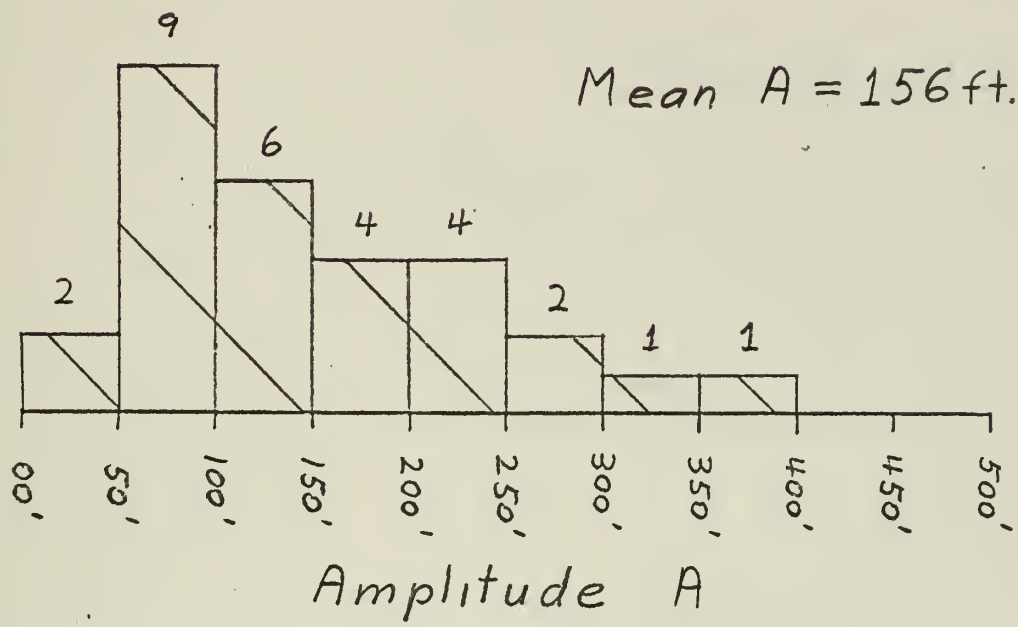




Figure 9

Wave No. 2, Latitude 40° North



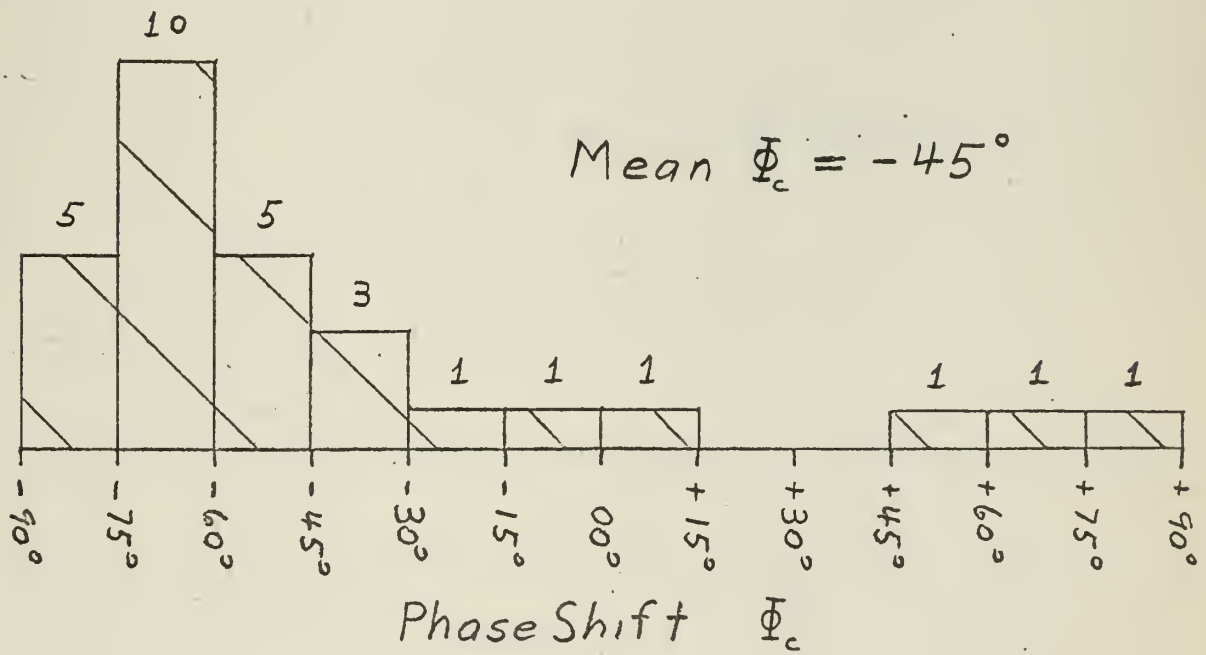
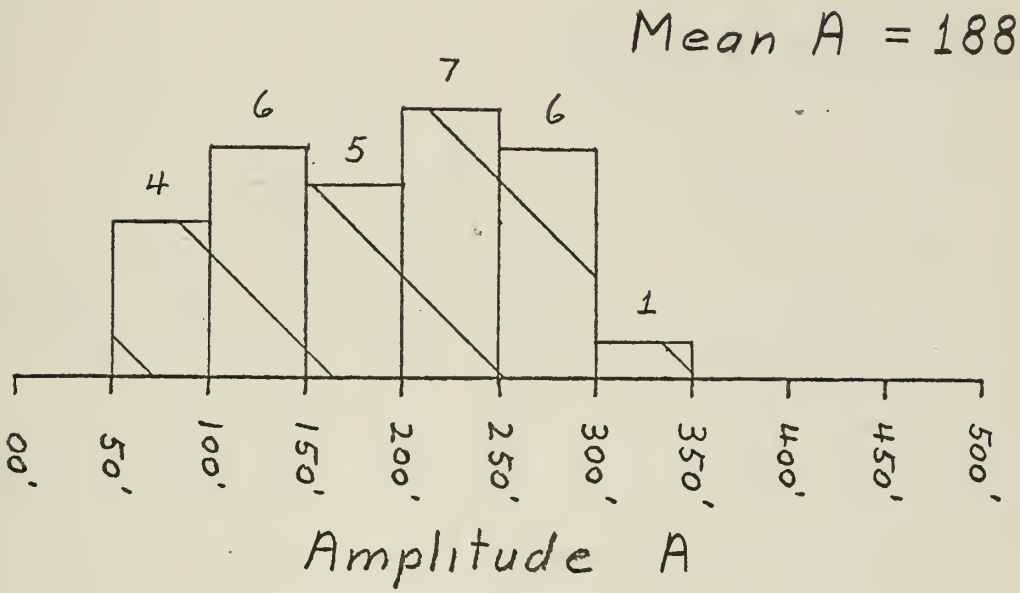
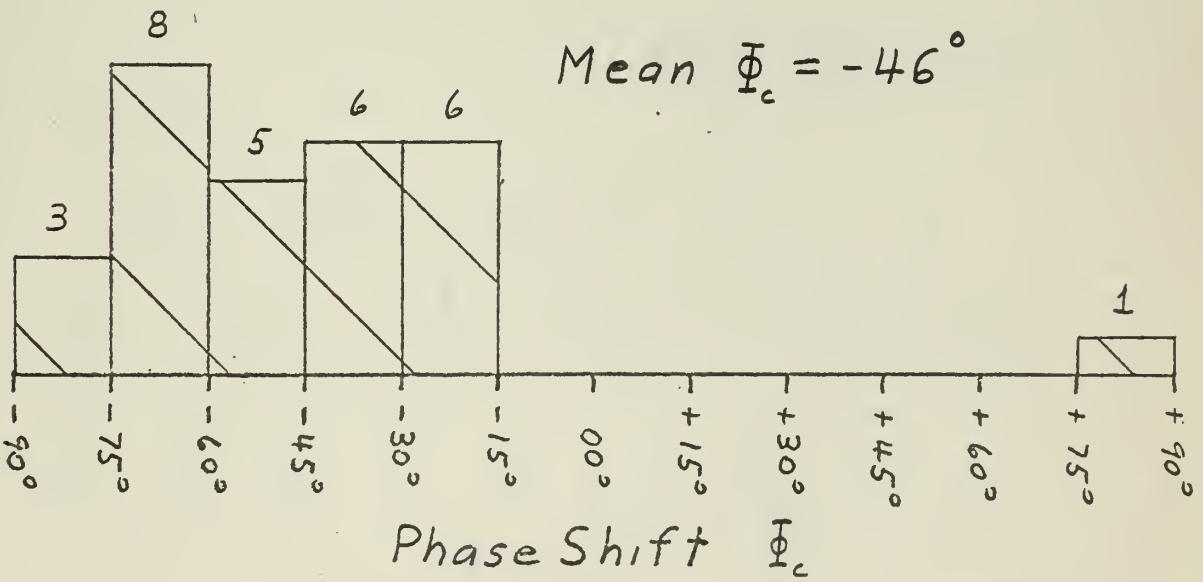




Figure 10

Wave No. 2, Latitude 50° North





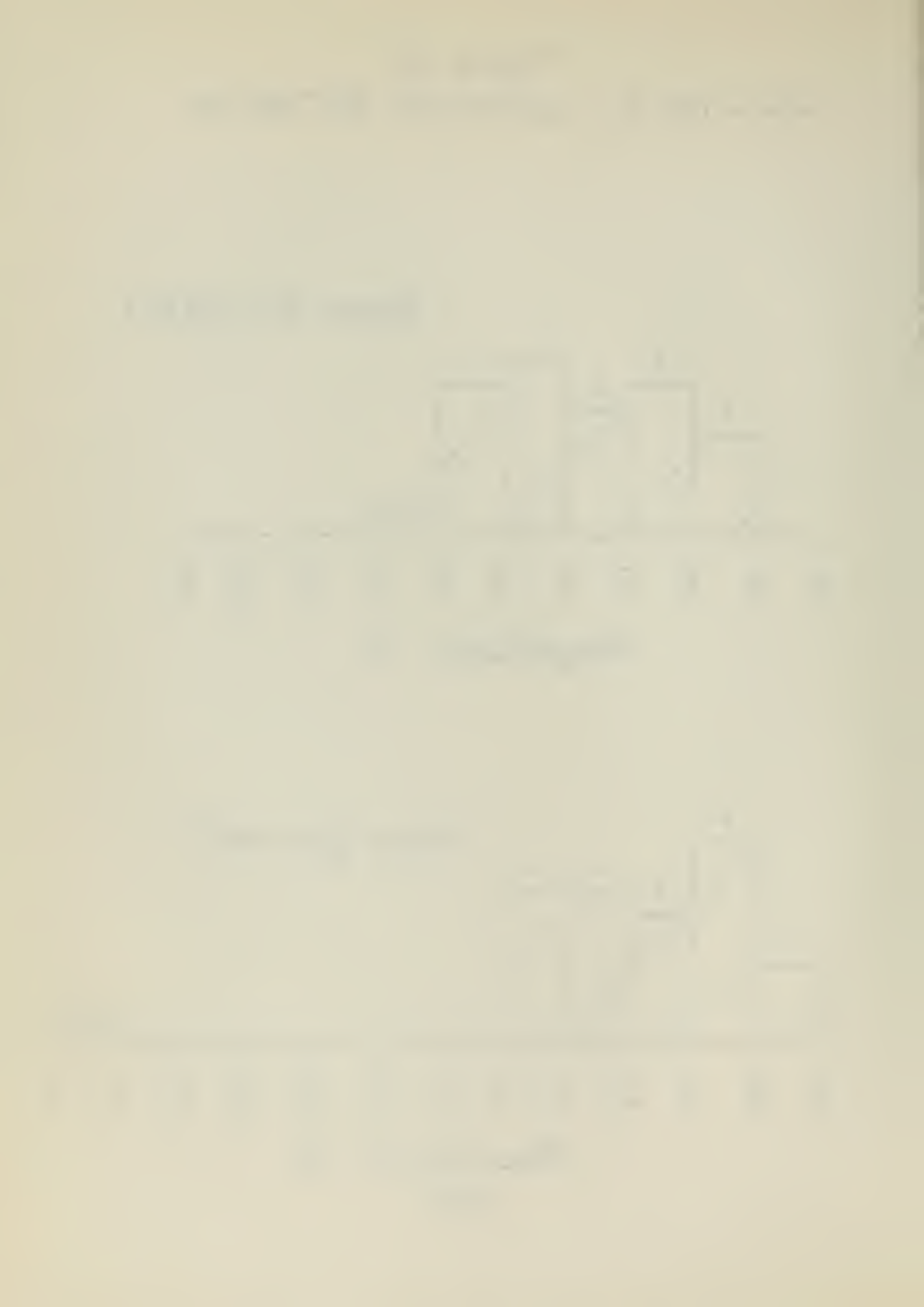
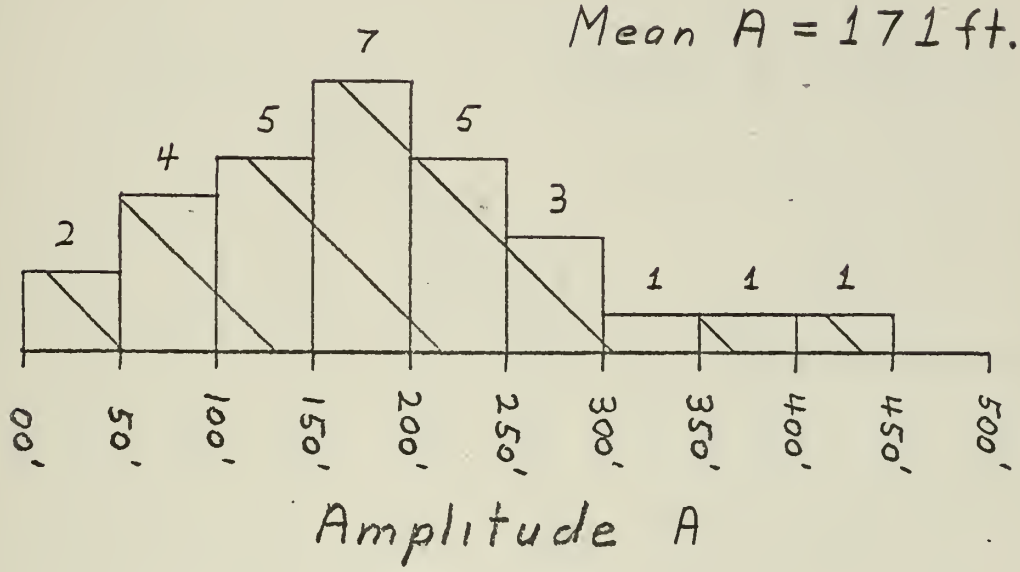


Figure 11

Wave No. 2, Latitude 60° North



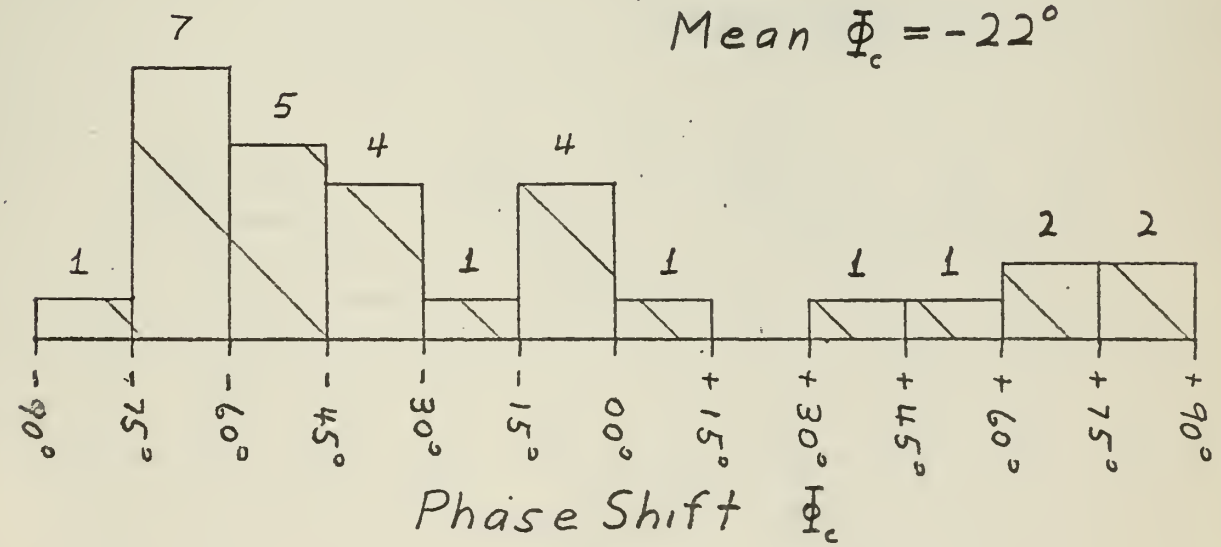
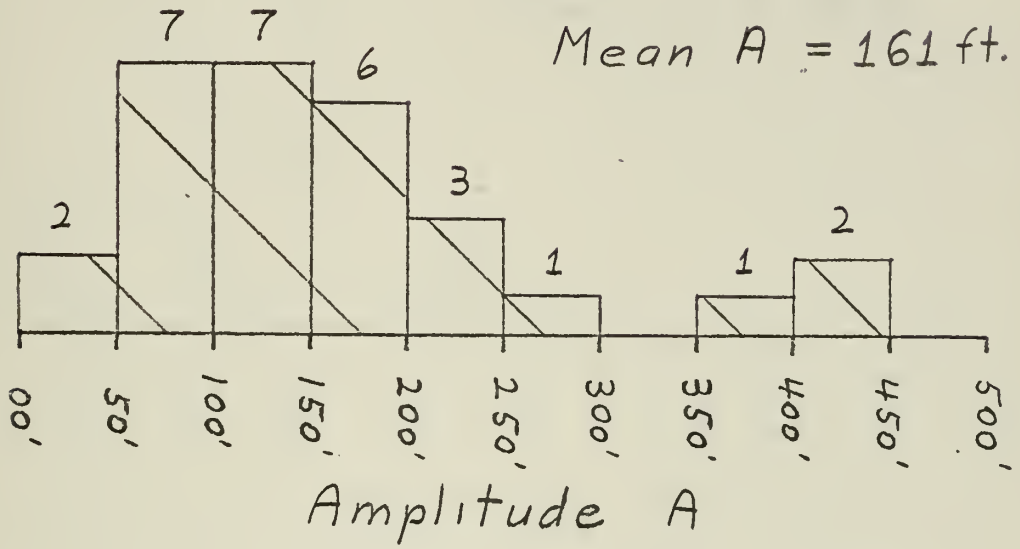




Figure 12

Wave No. 2, Latitude 70° North



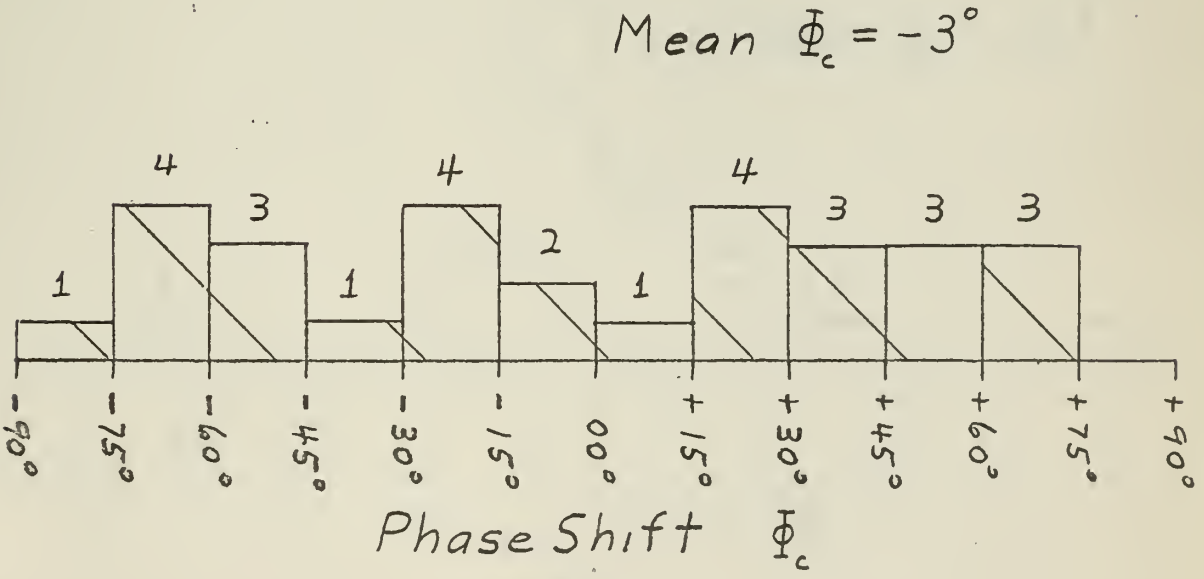
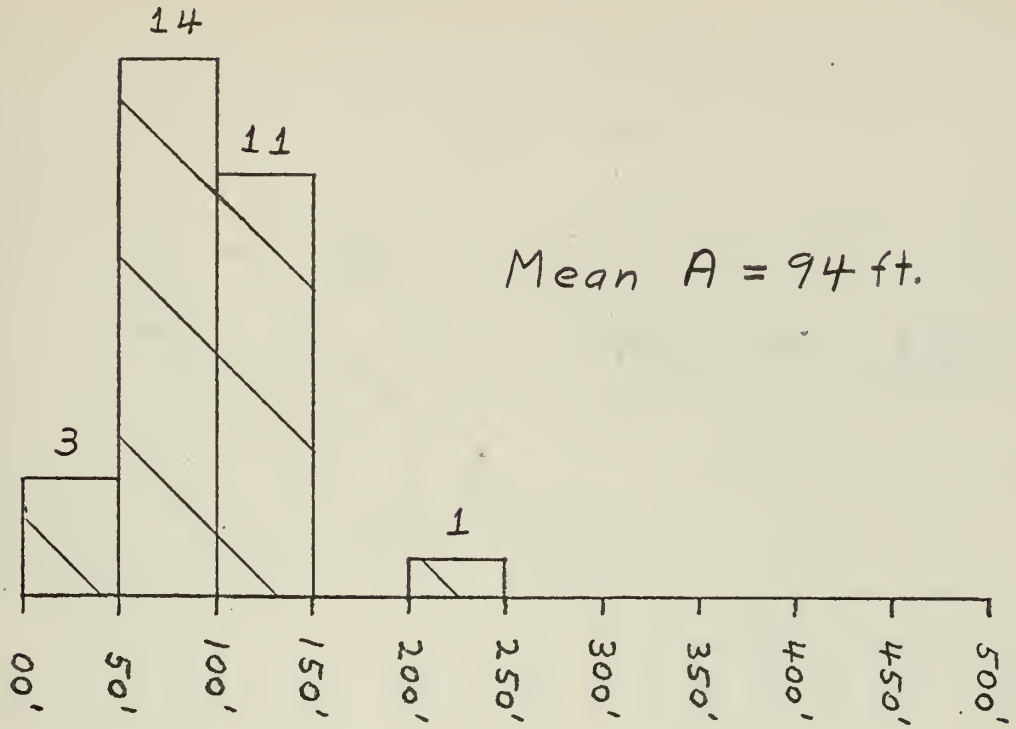


Figure 13

Wave No. 3, Latitude 20° North



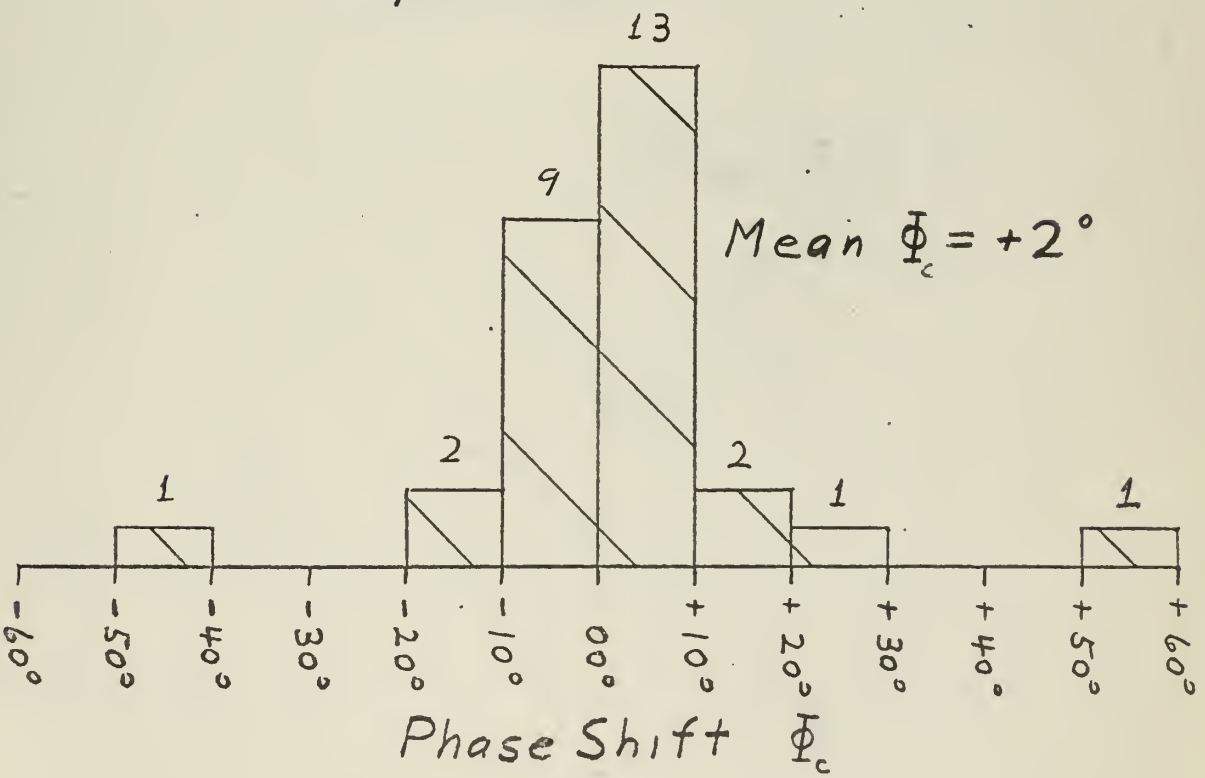
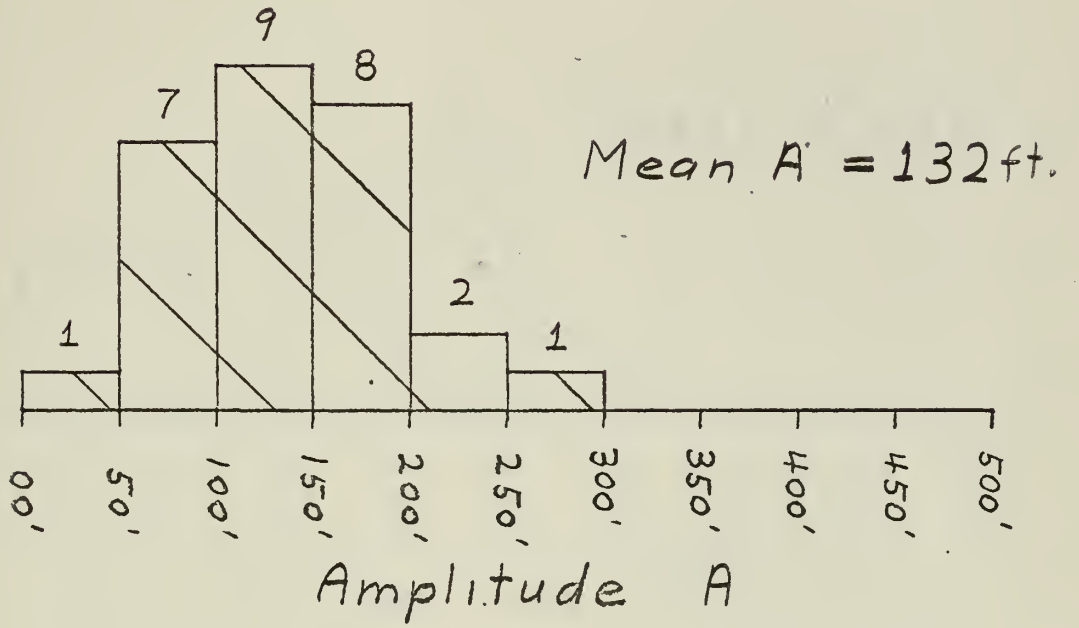


Figure 14

Wave No. 3, Latitude 30° North



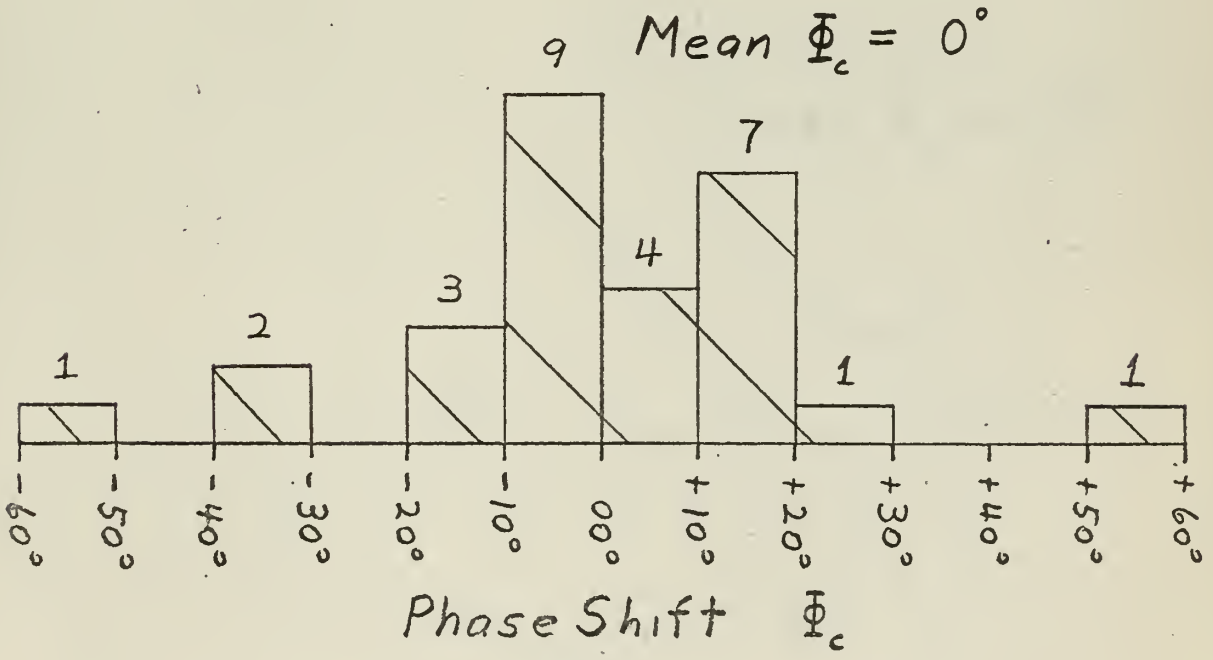
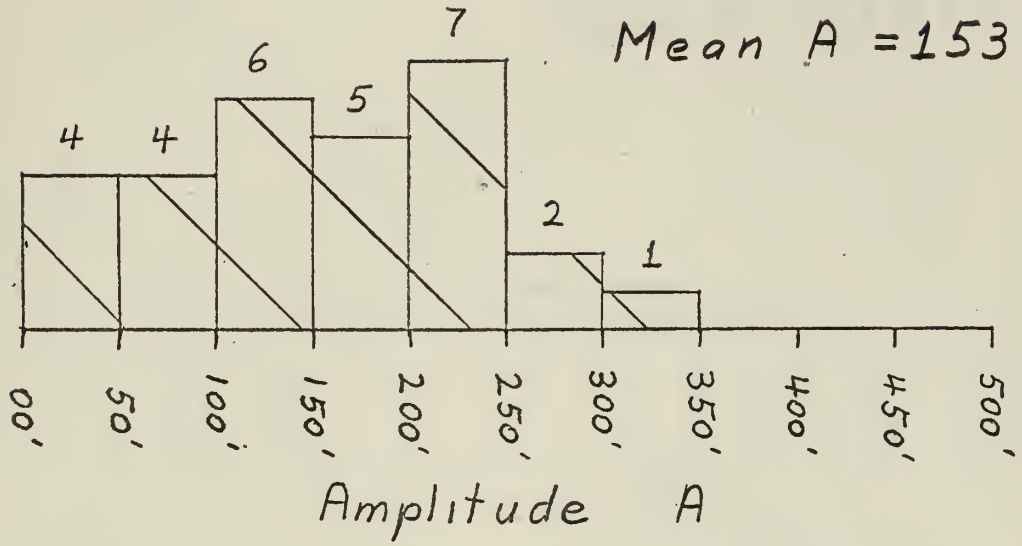




Figure 15

Wave No. 3, Latitude 40° North



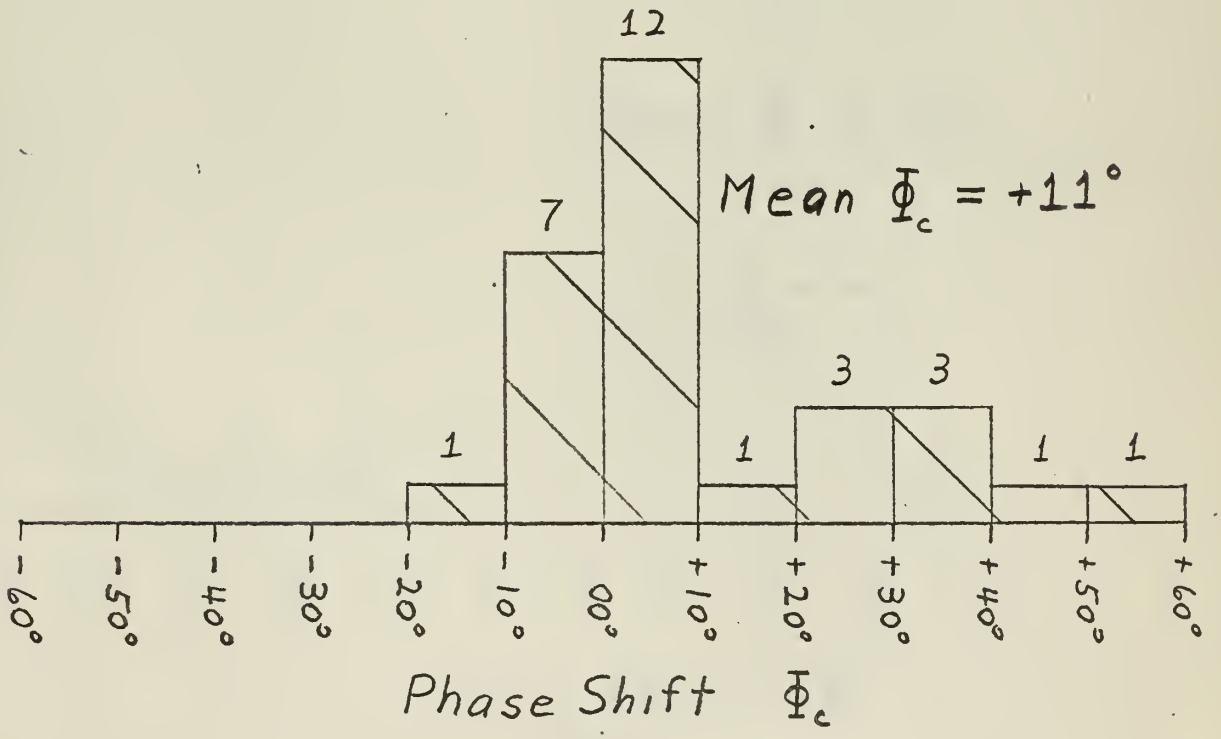
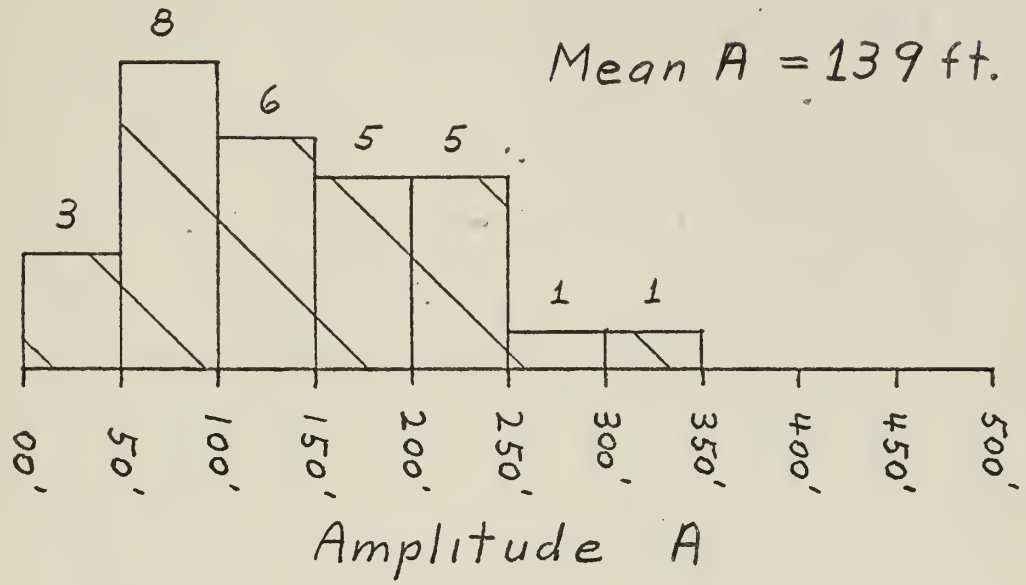




Figure 16

Wave No. 3, Latitude 50° North



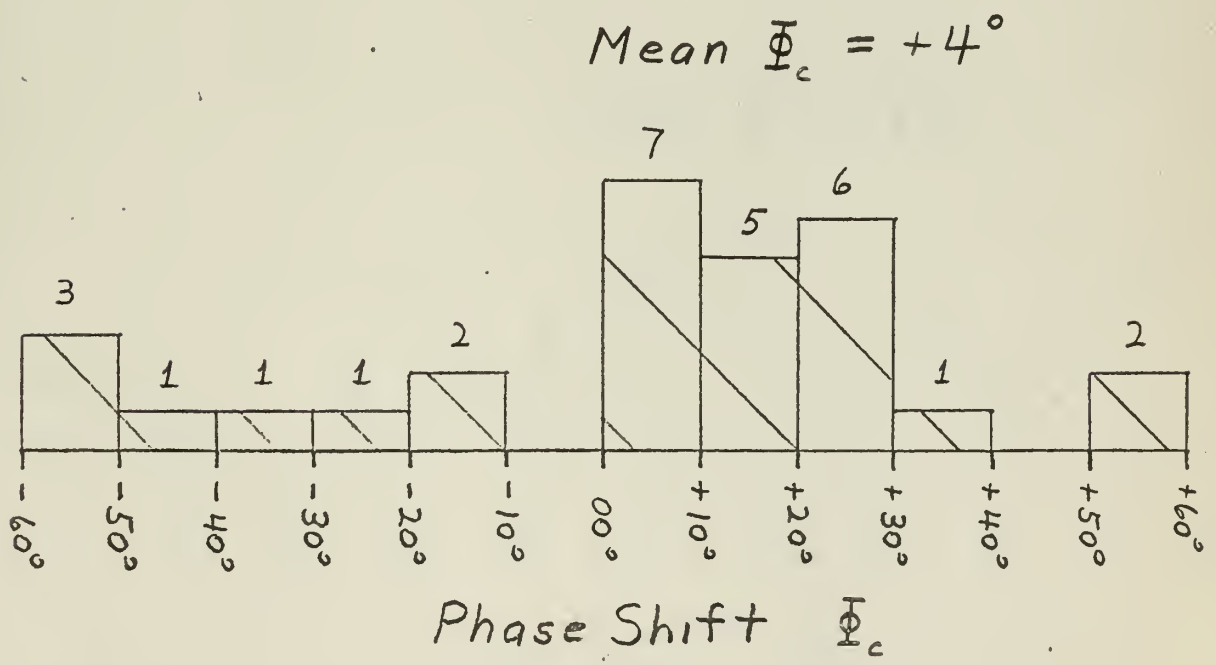
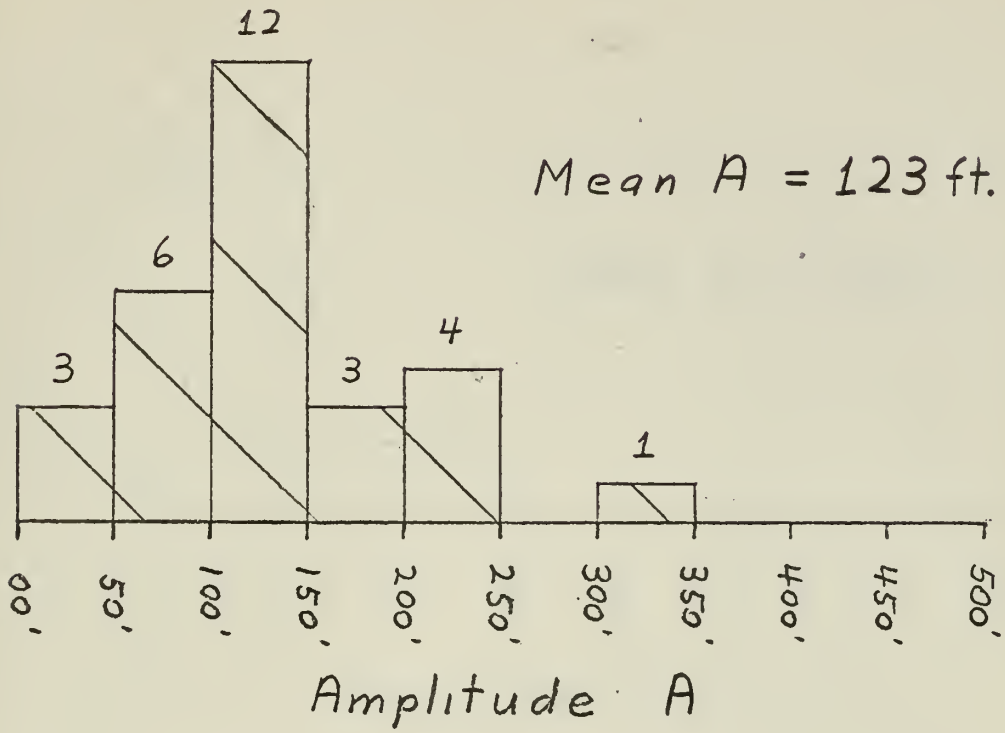


Figure 17

Wave No. 3, Latitude 60° North



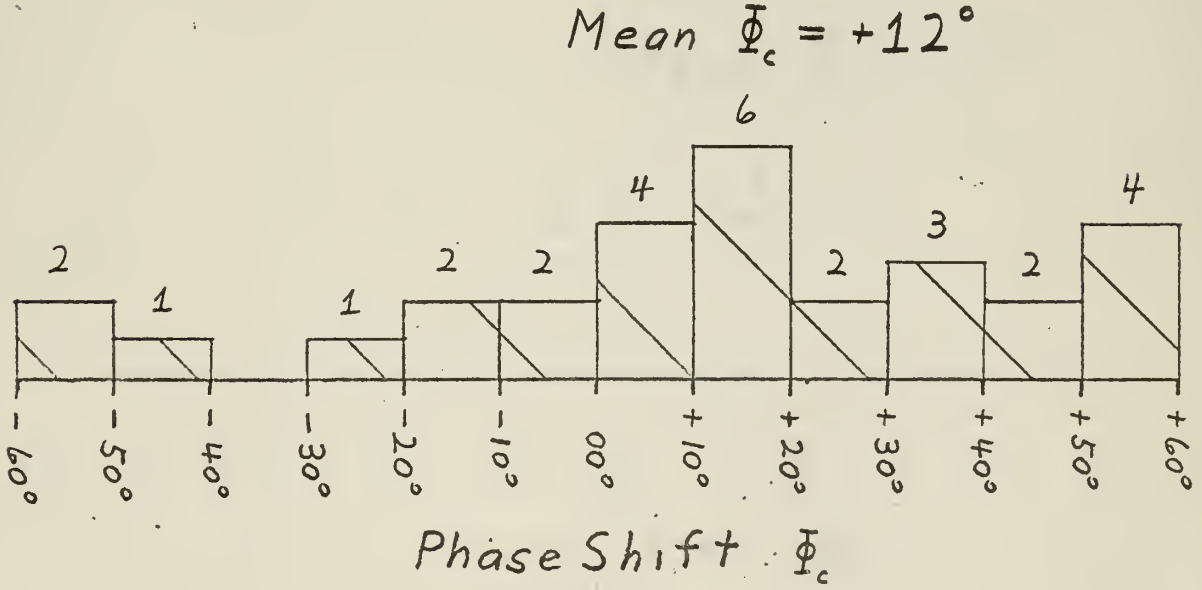
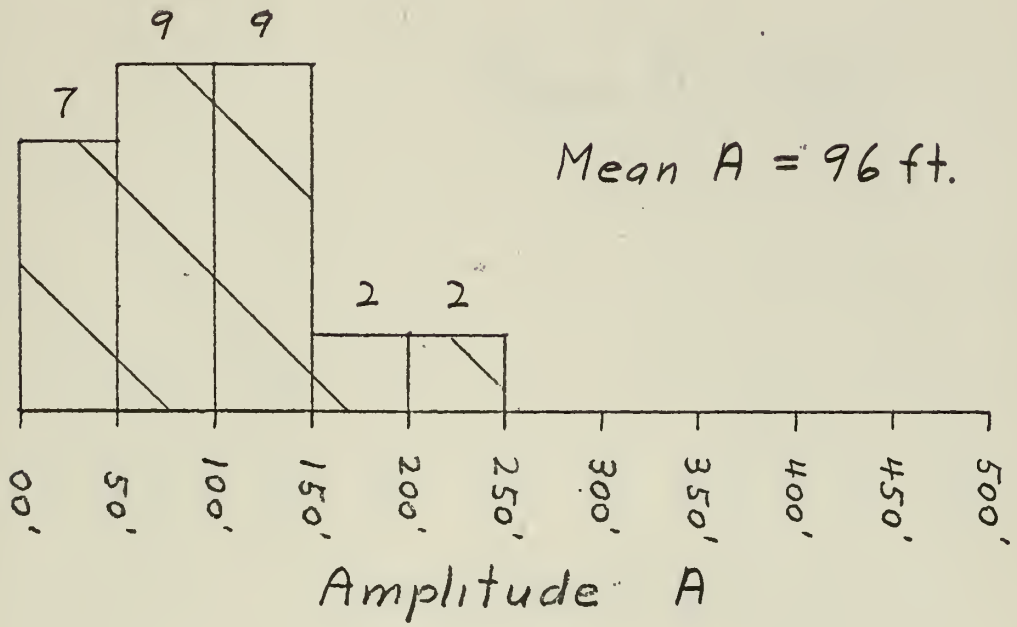




Figure 18

Wave No. 3, Latitude 70° North



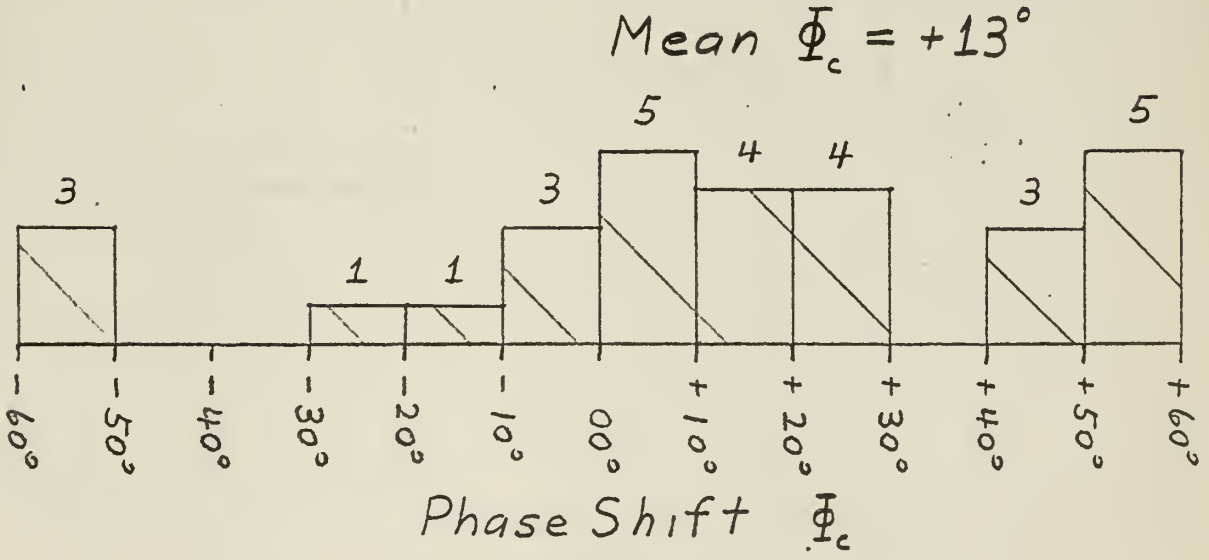
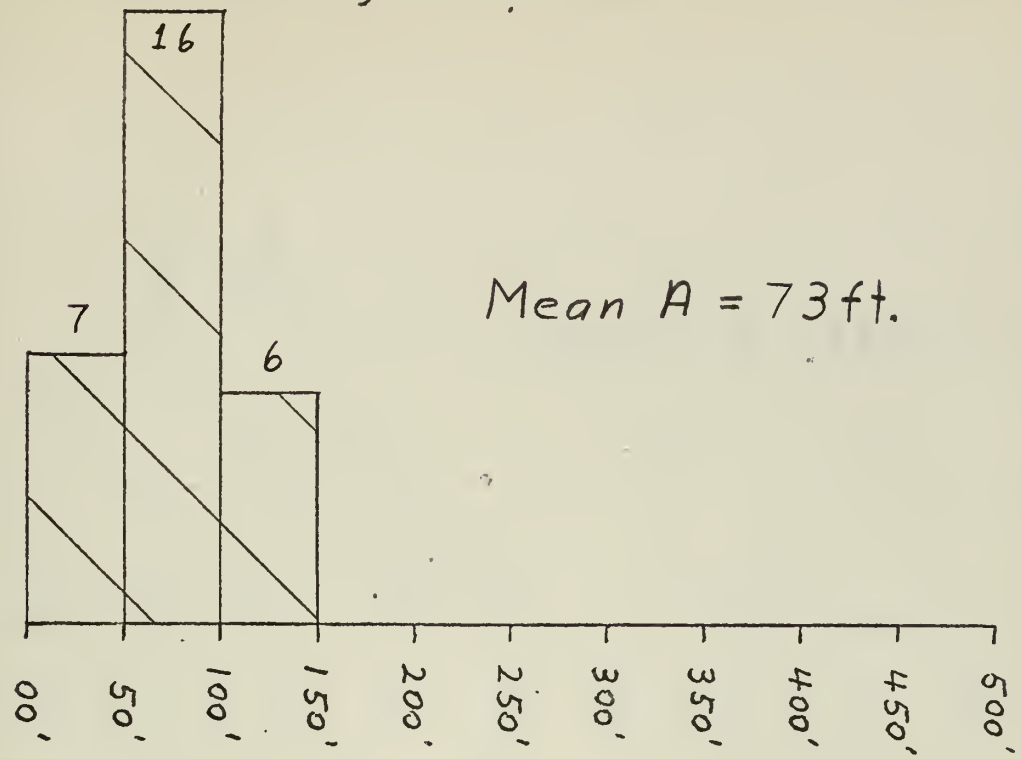




Figure 19

Wave No. 4, Latitude 20° North



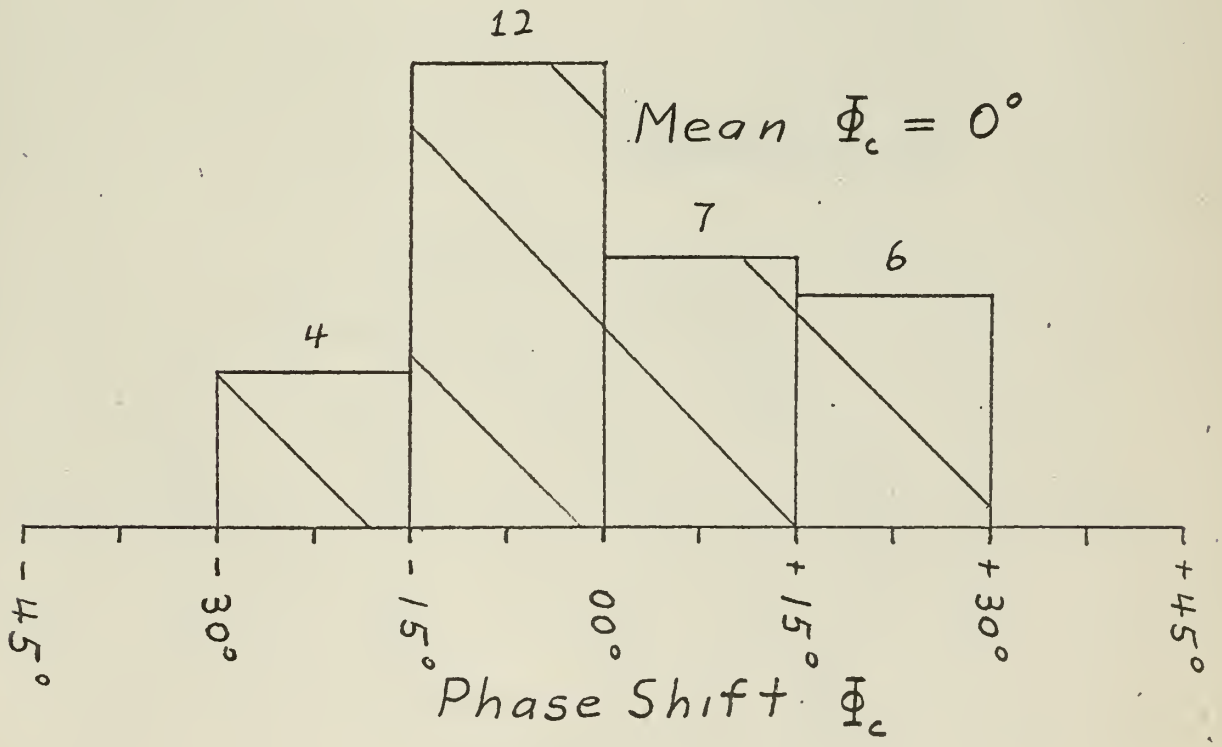
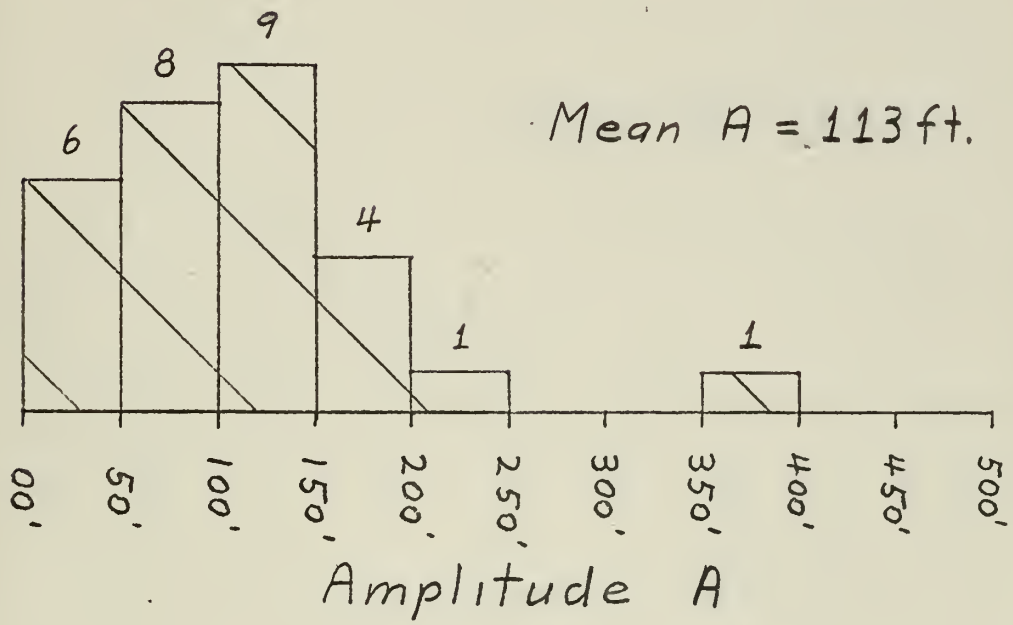




Figure 20

Wave No. 4, Latitude 30° North



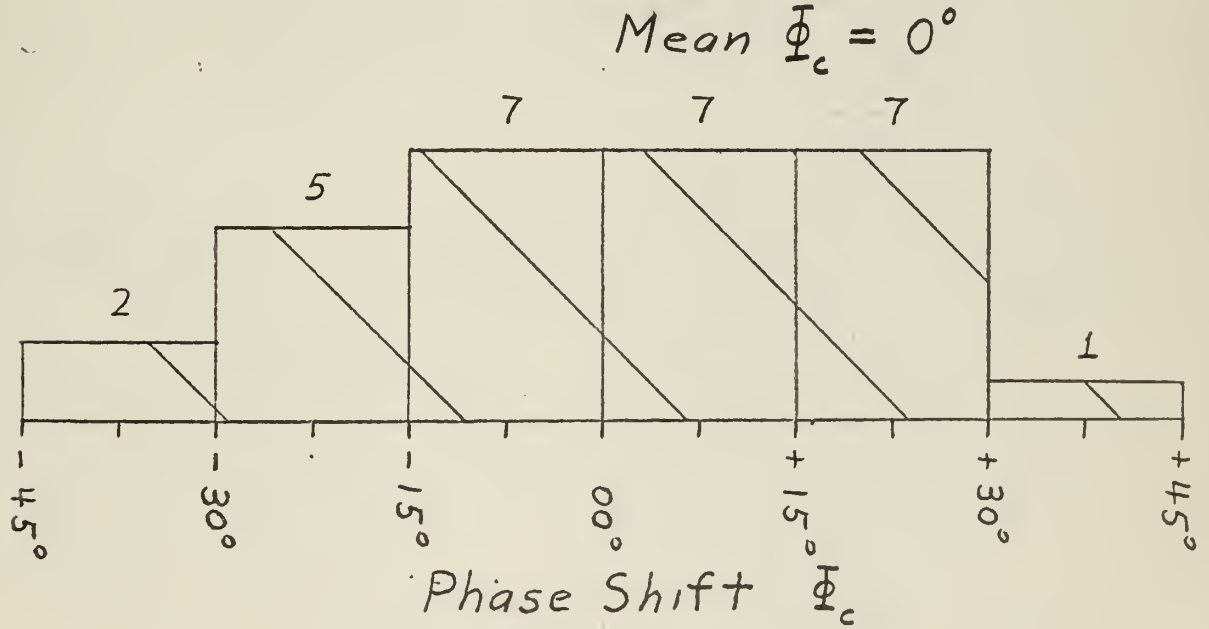
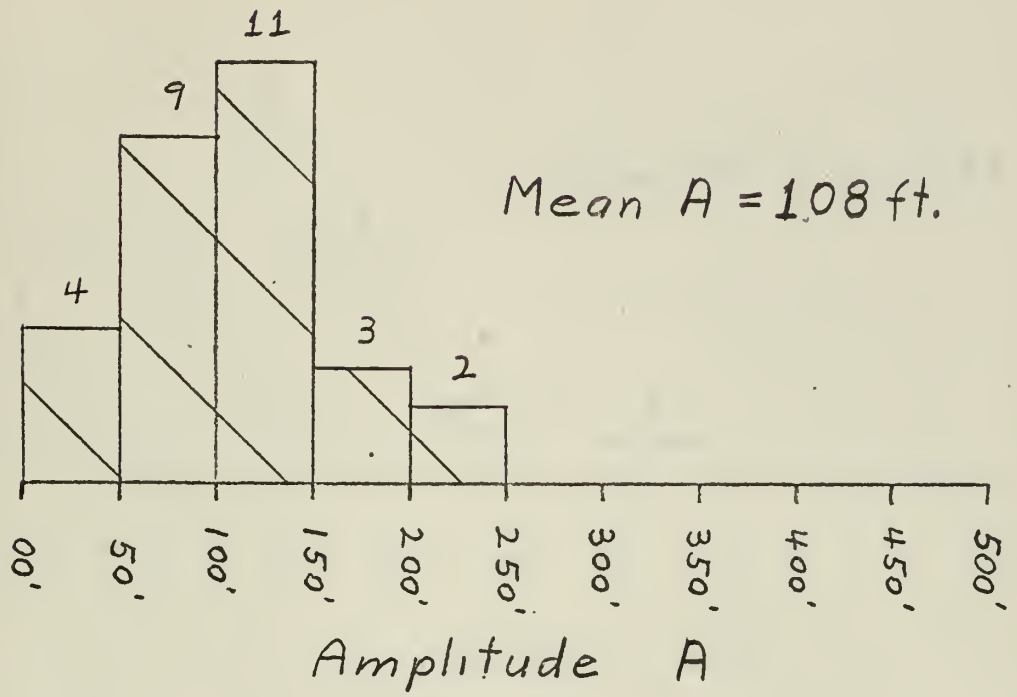
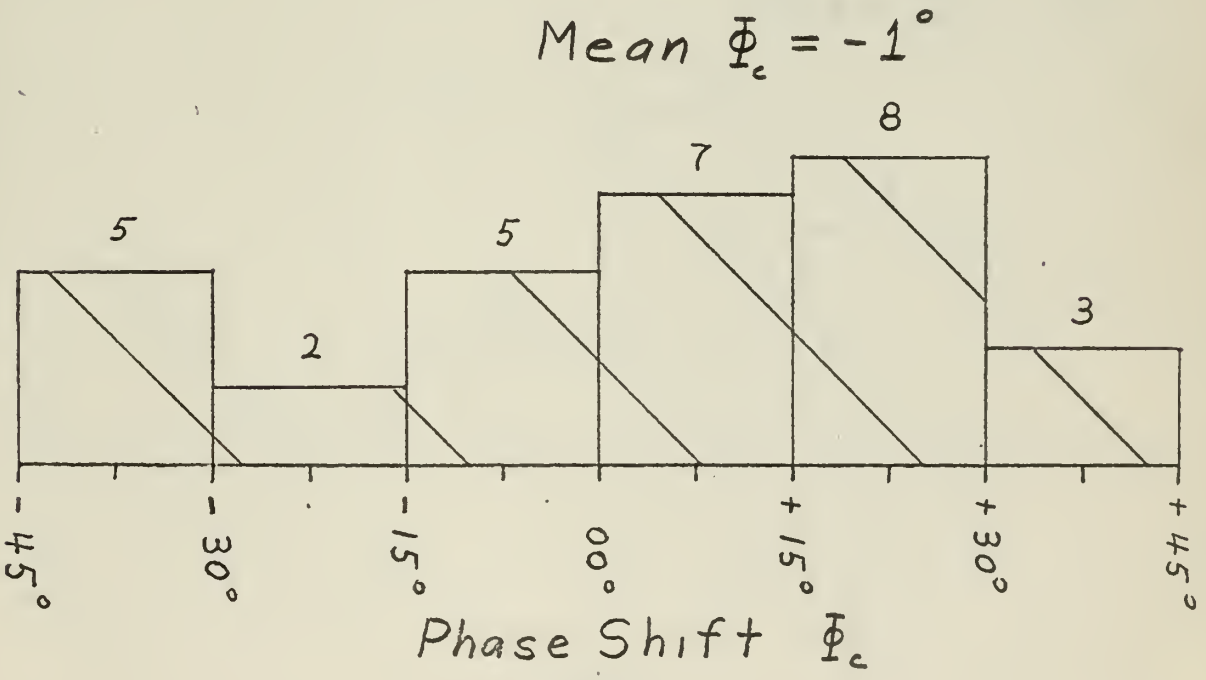




Figure 21

Wave No. 4, Latitude 40° North





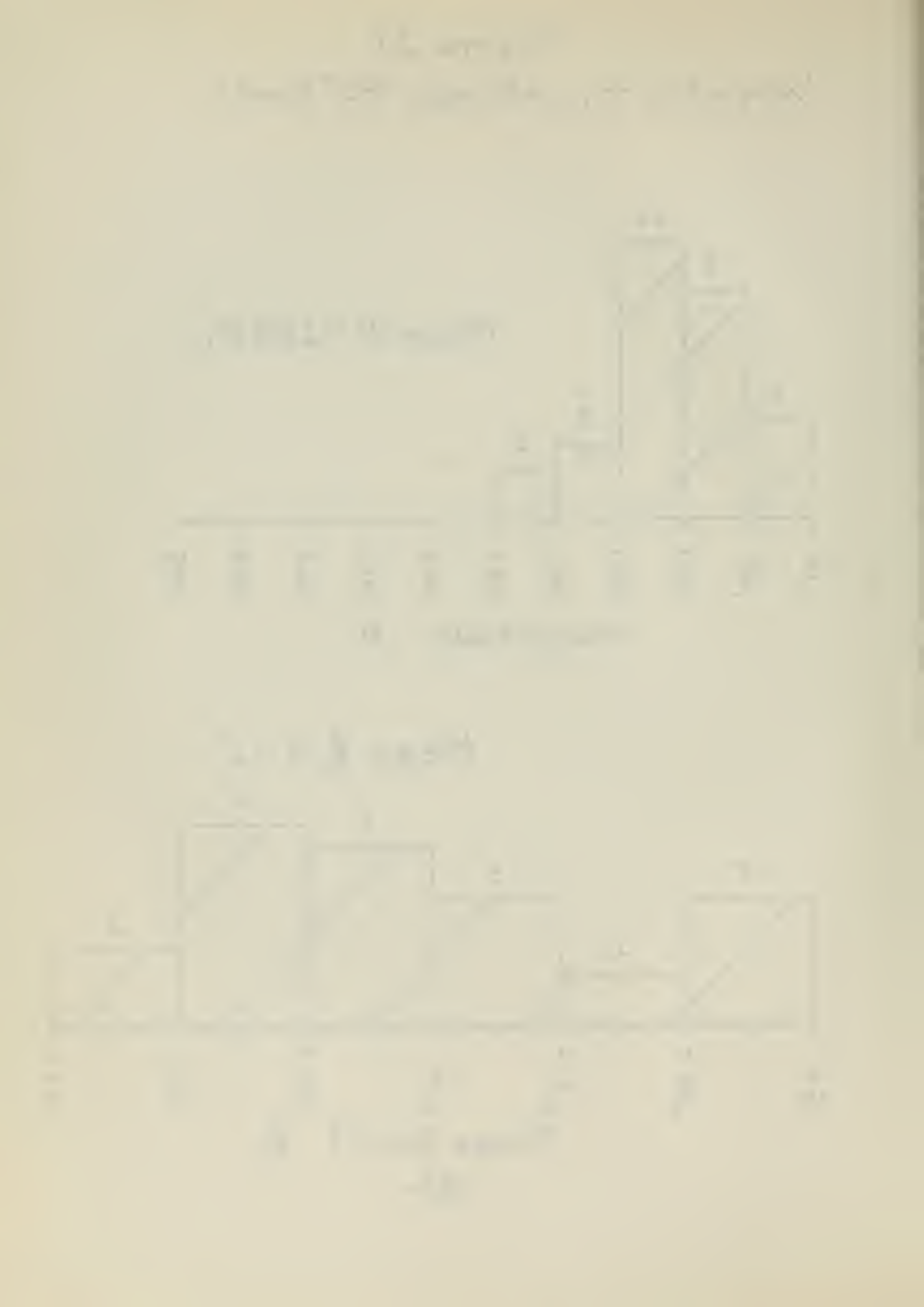
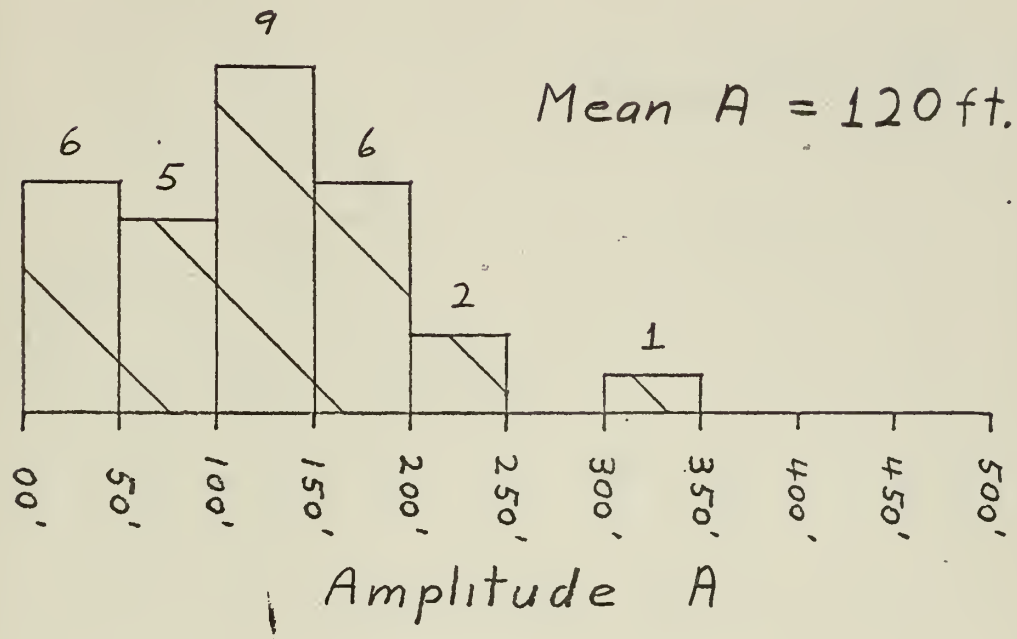
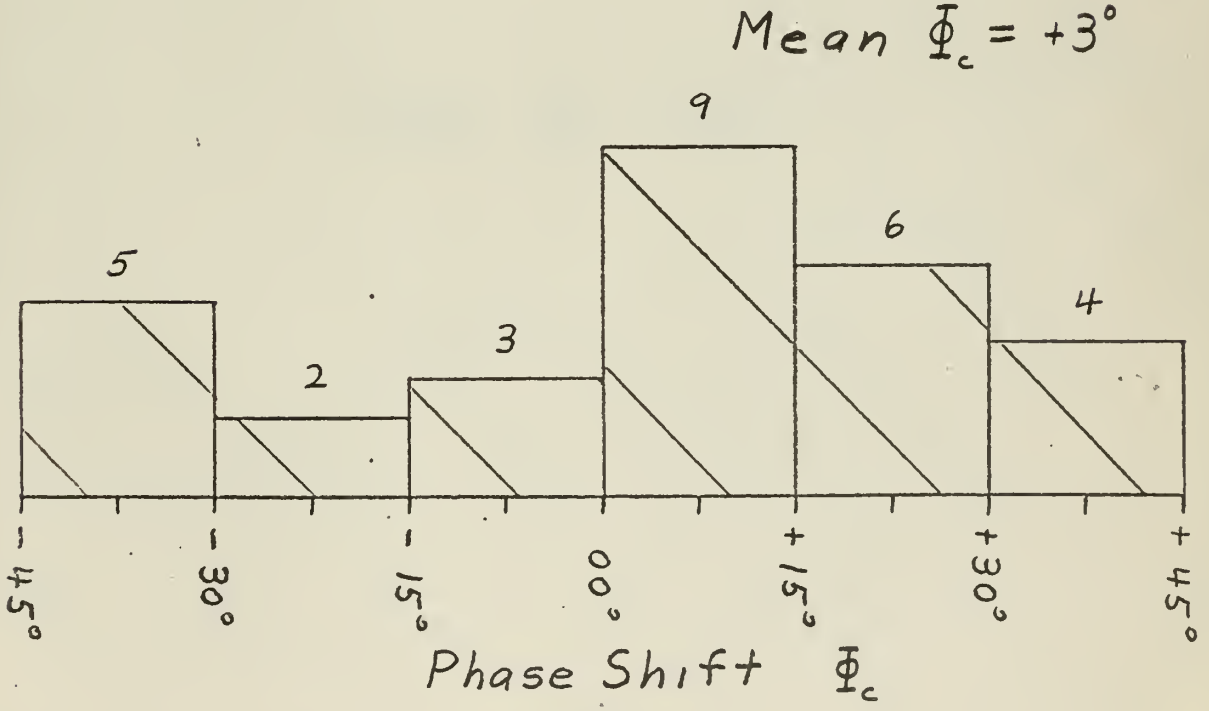


Figure 22

Wave No. 4, Latitude 50° North





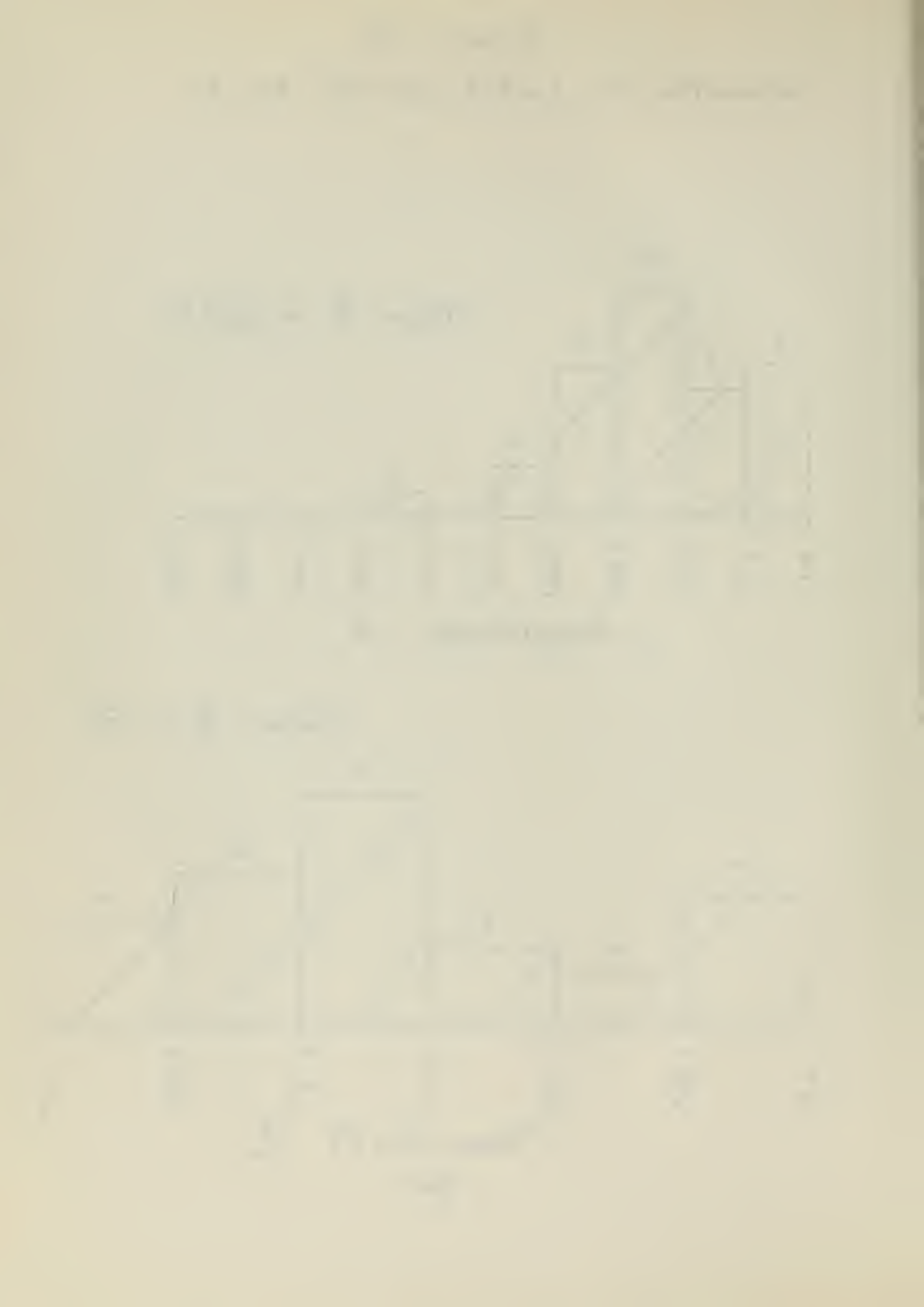
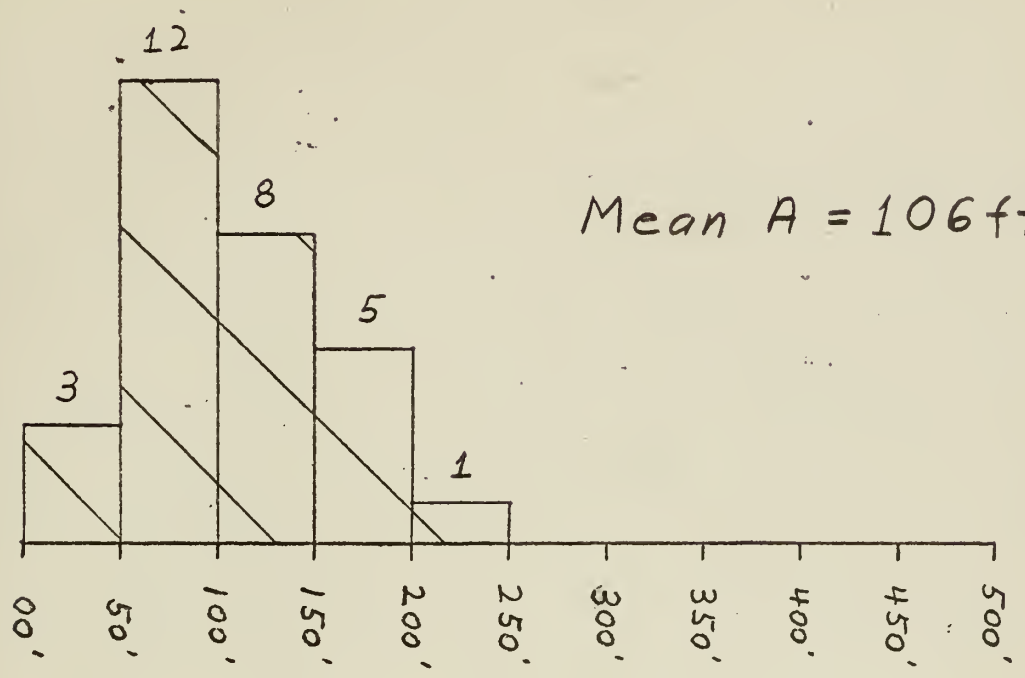
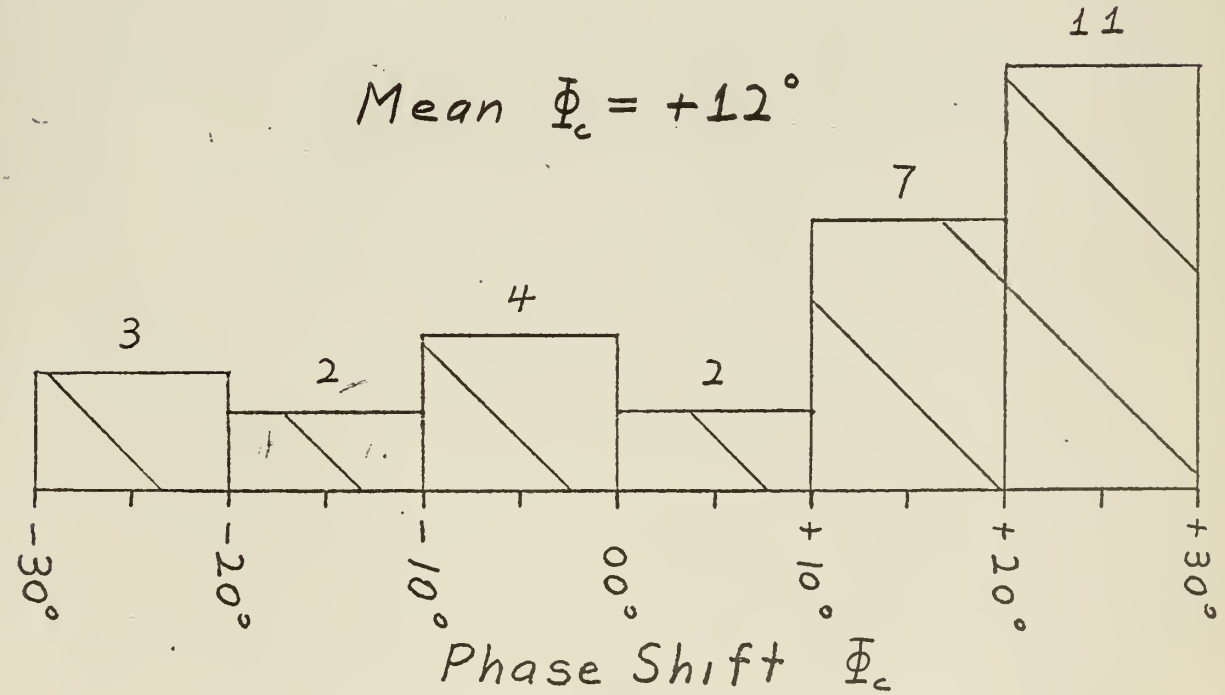


Figure 23

Wave No. 6, Latitude 50° North







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